Optimal Erosion Management on Developed Barrier Island Beaches

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Introduction

Coastal communities are threatened by natural hazards. Unremitting waves drive beach sediments along the shoreface (Cowell, Roy, and Jones 1995; Komar 1998). Sediment flux results in various patterns of erosion and accretion, with an overwhelming majority (80 to 90 percent) of sites in the eastern U.S. exhibiting net erosion in recent decades (Galgano and Douglas 2000). Erosion can be exacerbated by coastal storms. Climate change threatens to increase the intensity of storms and raise sea level 9 to 88 centimeters over the next century (IPCC 2001). Predictions for the U.S. suggest that 25 percent of homes within 500 feet of the coast could be lost to erosion in the next 60 years, at a potential cost of $530 million dollars each year (Heinz Center 2000). Managing coastal erosion impacts the quality of coastal natural resources. The economies of many coastal towns are dependent upon the appeal of beach and coastal resources, as their economic development has been primarily tied to the demand for beach and other forms of coastal recreation. To date, there has been very limited research on comprehensive management of barrier island beaches in response to erosion and sea level rise.[1]

The purpose of this research is to develop and implement an empirical model for optimal management of barrier island beach resources when the hinterland is developed. The model employs beach replenishment (addition of sediment) as a management control that can be used to manipulate beach width. The model can be used to characterize optimal beach width and provides a framework for planning beach erosion control projects under constant sea level and a constant erosion rate (*short term*) as well as increasing sea level and erosion (*long term*). Under some simplifying assumptions, the conceptual framework can identify the time horizon of management responses under sea level rise. An objective of this research is to explore whether active management (specifically, beach replenishment)[2] might be economically justified in the foreseeable future, or if passive management (shoreline retreat—i.e. letting erosion proceed unabated) is likely to become optimal in the long run. In the event of the latter, this

[1] In the only work to address this problem rigorously, Smith, Slott, and Murray (2007) cast beach nourishment as an optimal rotation problem. Their model provides theoretical insight on the problem of beach erosion management.

[2] Active management may also include shoreline armoring—the construction of large-scale protective devices on the shoreline. This research examines only beach replenishment. Shoreline armoring generally degrades overall beach quality, and is illegal in a number of coastal states (e.g. Maine and North Carolina).

research agenda aims to estimate the timing of a shift in management regimes and explore factors that influence this shift.

The long-run application of this model differs from conventional approaches to coastal protection by focusing on the stream of services derived from barrier beaches, rather than the value of threatened property at some future time. The received literature that considers coastal protection largely ignores external costs of such schemes. Structural fortification of the shoreline protects threatened property, but imposes external costs on beach resources. The expected value of threatened property is appropriate as a primary decision criterion if the value of beach resources is a small portion of the total economic value associated with a site (e.g. large coastal cities). The approach is less appropriate for most barrier islands, for which the value of beach resources can be a significant portion of total economic value, not only locally but nationally as well.[3] The methods utilized herein place beach resources at the focal point of the analysis, and are designed to address the problem of barrier island management in response to short term erosion problems and the long term problem of sea level rise.

An analytical model is developed that characterizes the optimal management response to erosion on barrier islands. The model focuses only on the average beach profile, and thus does not consider the distribution of beach quality along the shore or the alongshore dynamics that influence this distribution. This simplification is made for analytical tractability. The resource problem is decay of beach width, a dynamic process which is modeled as deterministic (but the extension to a random process is reasonably straightforward). Beach replenishment is introduced as a control variable that counteracts the natural erosive tendency. Dynamic optimality conditions are derived and discussed. The model is used to characterize optimal management under a constant erosion rate.

Application of the model produces estimates of the optimal schedule of beach replenishment operations for a specific coastline in the southeastern U.S., allowing for a corner solution at any point in time (no beach replenishment). Future applications of the model will examine whether beach replenishment is a tenable management practice in the long run, given assumptions about sea level rise and costs and benefits. A termination of

[3] Beaches are a leading U.S. tourist destination. Approximately 180 million "person-visits" are taken annually, and tourism in coastal areas accounts for 85 percent of U.S. tourism revenue (Houston 1996).

beach replenishment in the long run implies a policy of shoreline retreat, which would entail gradual migration of barrier islands and associated losses of property and infrastructure. A primary goal of this research is estimation of the optimal timing of such a transition. Information on the optimal timeline of shoreline retreat could be instrumental in allowing the market value of threatened properties to properly adjust to the risk of sea level rise[4] and invaluable for coastal planning and investment purposes. This research does not consider the distribution of cost and benefits engendered by beach erosion control, or other risks (hurricane and flood damage) engendered through development on the coastal fringe. Lastly, the empirically-derived costs of beach replenishment reflect only engineering and planning outlays, ignoring potential environmental impacts such as damage to benthic communities buried by sand or destroying during dredging and pumping and sterility of the replenished beach with may affect beach organisms, sea birds, and sea turtles.

The Coastal Erosion Problem

The coastal environment is one of the most dynamic places on earth. The position and form of the coastline are influenced by the interaction of the ocean, atmosphere, and coastal landforms; the shore attains a dynamic equilibrium, determined by waves, wind, ocean currents, sediment supply, storms, and sea level. As such, the coast has never been a particularly stable environment. This instability is not obvious to the casual observer, however, because the changes are very gradual in some cases, and sporadic in others.

Barrier islands dominate the eastern and gulf coasts of the U.S. Most of these islands exhibit sandy beaches—a common characteristic of the dynamic coastal equilibrium. Wave energy dissipates as waves strike land, and fine sediments can be deposited on the shoreface. This process gives rise to the sandy beach, an environmental resource often of considerable economic value. Despite the inherent instability of the coastline, a natural beach usually persists, albeit in possibly different forms and locations.

Coastal erosion is the loss of sediment, resulting in a recession of the shoreline. Low-angle waves (less than 45 degrees) drive sediment along the shoreface (Cowell,

[4] As Yohe, Neumann, and Ameden (1995) recognize, the trajectory of threatened property values is largely dependent upon the perception of the likelihood of abandonment.

Roy, and Jones 1995; Komar 1998), typically resulting in net erosion over time. Beaches can be decimated by energetic waves associated with coastal storms. These storms can move a great deal of beach sand offshore, but most of the sand subsequently returns to the beach in the majority of cases. This oscillating process, in fact, protects land behind the beach from the direct attack of storm waves, but can create erosion problems in the short term as the position of the shoreline fluctuates.

The stochastic oscillation of the shoreline in response to coastal weather patterns is tied to a baseline sea level, which is currently rising at an average of 1 – 2 millimeters per year (Edgerton 1991).[5] With a rise in sea level, undeveloped barrier islands move landward by rolling over, as sand is transported from the ocean to the land side; the beach, being an equilibrium characteristic, will tend to migrate landward over time as the island recedes (Dean and Maurmeyer 1983; Leatherman 1988; Pilkey and Dixon 1996). Developed barrier islands differ from their undeveloped counterparts in that they are anchored to a specific location by the existence of infrastructure, housing, businesses, etc. Since these islands are not allowed to migrate, they can become increasingly threatened by inundation and increased storm wave heights.

The loss of beach sediments reduces beach area, can diminish sand dunes, and may ultimately threaten property and infrastructure on the shoreline. Loss of beach area can affect recreational use, through both reduced capacity to support recreational and leisure activities and through diminished aesthetics that impact economic value of users (Bell 1986; Parsons, Massey, and Tomasi 2000). Erosion of the beach and loss of sand dunes exposes coastal development to chronic wave-driven erosion and can increase damage during coastal storms by exposing coastal properties to greater storm surge and wind. We focus on beach width as a measure of beach quality that can be manipulated by the coastal planner through beach replenishment — dredging or trucking sand deposits from other locations and them pumping onto the beach face.

These facts characterize a difficult public policy problem. Beaches are a source of recreational and aesthetic value. Development on the coastal fringe facilitates access to beaches and provides for enjoyment of scenic amenities, but limits the shoreline's ability to evolve in response to coastal hazards. Beach replenishment can ameliorate the

[5] The relative rate of sea level rise also depends upon the local rate of land subsidence.

effects of wave-driven erosion and sea level rise, but the improvements are transitory and the costs are considerable. The costs of beach replenishment have become a critical issue as federal support for shoreline management projects has declined in recent years.

Optimal beach management reflects a balancing of the economic benefits and costs of remedial actions and incorporates the dynamic effects of management decisions on relevant beach services. In the short term, one can derive an optimal time path of the management and state variables. In the long term, sea level rise increases erosive pressure on the shoreline and may render some settlements indefensible. The optimal management strategy in the long term depends on the degree of erosive pressure (i.e. sea level rise), how this affects management costs, and the benefits of preserving the current shoreline. Existing literature on coastal protection has focused on the value of coastal property that is threatened by sea level rise. As Yohe, Neumann, and Ameden (1995) point out, with full information on the risks of sea level rise, market depreciation over 30 years time could drive the value of this property to zero. While this outcome is complicated by uncertainty regarding sea level rise and the inherent lack of reliability in a commitment to abandon property, it can clearly be problematic to rely on such a subjective decision criterion for coastal policy making.

Economists' Thinking on Coastal Erosion

There are two branches of literature that have examined the economics of coastal erosion management—the first primarily considering the short run problem of efficient management in response to chronic wave-driven erosion and the second addressing the long run problem of coastal protection under sea level rise.

The existing literature on beach erosion management in the short term is primarily static in nature and tends to focus on limited impacts of erosion control. Bell (1986) and Silberman and Klock (1988) estimate the recreational benefits of beach replenishment. Bell (1986) estimates the optimal square-footage of beach space per beach user and compares constant benefits of maintaining optimal beach area with beach replenishment cost estimates (finding rather large benefit-cost ratios). Silberman and Klock (1988) use a split-sample stated preference survey to estimate differential WTP for beach recreation before and after a beach replenishment project; they find a stronger effect on visitation than on benefits per trip, but positive net benefits for beach replenishment overall. Edwards and Gable (1991) and Pompe and Rhinehart (1995) employ hedonic property price models to focus on coastal homeowners' preferences for beach quality. Edwards and Gable argue that proximity to the beach reveals implicit savings in travel cost that reflect household preferences for beach recreation, and they attempt to identify demand for distance as a measure of economic welfare. Pompe and Rhinehart examine the value of proximity and beach width in an attempt to disentangle protective from recreational benefits.

Other authors have addressed shoreline retreat. Parsons and Powell (2001) provide an estimate of the costs of shoreline retreat on property owners on the Delaware coast and compare this to beach replenishment cost estimates. Their findings suggest that the costs of replenishment are less than the adjusted value of houses that would be lost over the next 50 years. Landry, Keeler, and Kriesel (2003) compare beach replenishment, shoreline armoring, and shoreline retreat over 25 years, taking account of property effects and recreational benefits accruing to both beach visitors and coastal homeowners. They find that the efficiency of active management depends upon the erosion rate and how management costs evolve over time (given sea level rise and changes in resource stocks and technology).

Smith, Slott, and Murray (2007) offer the first theoretical model of beach erosion management, employing a Faustmann-like framework in which the optimal rotation time between replenishment operations is the object of choice in a time autonomous (i.e. short term) setting. Unlike the model offered below, they employ a non-linear erosion rate to account for adjustment to equilibrium profile, and show the optimal rotation time can increase or decrease in response to an increase in variable costs of replenishment (i.e. sand costs) depending upon whether the rate of decay of nourishment sand exceeds the discount rate. Further, they show that (*ceteris paribus*) optimal rotation time: i) decreases when benefits of beach preservation increase (as reflecting in beachfront property values), ii) increases with higher fixed costs of replenishment, iii) decreases with a higher erosion rate, and iv) decreases for a higher discount rate.

We consider next studies of coastal protection under sea level rise. Titus et al. (1991) estimate the nationwide costs of protecting developed coastal lands and the losses associated with undeveloped lowlands and wetlands, for a range of sea level rise scenarios.[6] Their cost estimates for protecting developed coastal land from a one-half to one meter rise in sea level are between $55 and $305 billion and that the United States could lose between 20 and 69 percent of its coastal wetlands. They conclude that the environmental effects associated with lost wetlands could be catastrophic, and recommend a gradual abandonment of undeveloped coastal lowlands in order to allow wetland migration. Their analysis presumes that all developed coastal land will be protected and presents a *positive* economic assessment of the costs associated with that scenario—roughly $2,000 per quarter-acre, assuming coastal development is confined to its present locations. Their estimated price tag of lot is an average figure, which does not allow for a determination of which areas should be protected.

Recognizing this, Yohe (1991b) offers a framework for a *normative* economic assessment of coastal protection schemes, utilizing a stochastic sea level rise trajectory and a corresponding trajectory of marginal property damages for Long Beach Island, NJ. He finds raising the island is the best course of action under gradual sea level rise; building a dike is preferred under accelerated sea level rise. But, he does not consider the impact of management on barrier island beaches. Yohe and Neumann (1995) and Yohe, Neumann, and Ameden (1995) explore coastal protection under different property value scenarios, such as property values increasing "business as usual" or property values depreciating in anticipation of sea level rise. Their simple dynamic optimization model posits the choice of beginning and terminating active management times as a function of the present value of benefits (property value trajectories) minus the present value of costs (management expenditures and property losses at terminal time). The latter study finds that Sullivan's Island (near Charleston, S.C.) should be protected immediately, through beach replenishment. In the case of this barrier island, the only real timing decision is deciding *when to stop protection*. This is precisely a question we intend to address.

[6] Their analysis assumes that raising barrier islands by pumping sand is the preferred response (while levees and pumping systems will protect developed portions of the mainland).

Theoretical Framework

Assume there is a time-dependent variable representing average beach width, q_t. This is our measure of beach quality. Initial beach width is taken as given, but subject to erosive force that reduces beach width in a deterministic manner. The coastal planner can augment beach width by adding sand to the beach.[7] The length of the beach is taken as given, and thus beach width determines beach area, which provides space for recreational and leisure activities for both visitors and local residents and contributes to the aesthetics of the coastal landscape. Beach width also provides protection from chronic erosion and high velocity waves, wind, and erosion associated with coastal storms.

Following Landry, Keeler, and Kriesel (2003), agents affected by beach management can be classified into two groups: beach visitors and coastal property owners. The former includes households that participate in recreational and leisure activities at the beach and are thus concerned about beach quality, but do not own a stake in island property. The latter are similarly concerned with beach quality, but are also concerned about maintaining their property, which can be threatened by chronic erosion, coastal storms, and sea level rise. Other beneficiaries of beach management could include prospective users, those concerned about preservation for future generations, or those who feel that beaches should be preserved due to intrinsic value. At this juncture, we ignore recreational user benefits[8] and draw on empirical results from Landry and Hindsley (2007) for measures of homeowner WTP for improved beach quality. We consider empirically derived measures of replenishment cost, but neglect possible environmental costs (reflecting damages to benthic or beach organisms) or external benefits or costs affecting adjacent communities.

The coastal planner's problem is to maximize the difference between total benefits and costs of management subject to an erosion constraint which describes how beach quality evolves over time. The coastal planner chooses the amount of beach replenishment to be conducted in each period. This problem is a non-renewable resource

[7] We assume that any additional sand is of a similar quality to the existing sand, so there are no other qualitative or aesthetic effects associated with beach replenishment.

[8] Recreational users may have many substitute beach sites available, and thus not be overly concerned about beach quality at any one particular site. Whitehead et al. (2008) find that recreation demand does respond to a 100 foot increase in beach width due to beach replenishment, but the effect is not statistically significant.

management problem, but differs from the conventional non-renewable problem because society benefits from preservation rather than extraction. The non-renewable resource exhibits a decaying tendency, and the management control represents a contribution to the level of resource quality that counters the tendency for decay.

A sustained corner solution implies a *de facto* policy of shoreline retreat in the long term. By "sustained corner solution" we mean a lack of control for a period of time sufficient to lead to significant diminution of beach resources and associated losses of property. Returns from beach quality are given by willingness to pay of property owners (and possibly visitors and other interested parties), and these preferences are taken as static. Since replenishment costs are expected to increase with sea level, one can define the point at which shoreline retreat becomes the optimal policy response by the balance of benefits and costs. If costs rise sufficiently, they will eclipse the benefits of preservation, thus triggering a policy shift.[9]

Using control theory, the management problem can thus be represented as:

$$\max_{n_t} \sum_{t=0}^{T-1} \eta^t \{WTP(q_t) - C(N_t)\} \qquad [1]$$

subject to $q_{t+1} - q_t = -\theta + \tau n_t$ [2]

$N_t = n_t \times l \qquad n_t \geq 0$ [3]

$q_{t=0} = q_0,\ q_T = free \qquad q_0 \geq 0$ [4]

where *WTP* reflects aggregate willingness-to-pay for beach quality level q_t (derived from a hedonic property model); $\eta^t = (1 + \delta)^{-t}$ is a discount factor; $C(N_t)$ represents the costs of beach replenishment, with N_t representing the total volume of sand (or "beach fill") added to the beach in period t; n_t is the volume of beach fill *per unit of beach length* (l) (i.e. if N is total sand volume for a project, $n=N/l$); τ is a parameter that converts sand volume to incremental beach width; $q_{t+1} - q_t$ describes the dynamic motion for beach width (bolstered by beach replenishment and naturally decaying at some rate θ); and q_0 is the

[9] Interestingly, barrier island beaches may be considered a renewable resource under a retreat scenario, assuming that island migration can keep pace with sea level rise. The problem as non-renewable is related to maintaining quality *and the present location* and thus preserving coastal developments as well.

initial beach quality condition. The terminal level of beach quality (q_T) is free, but could be specified as a specific value.

Equations [1] through [4] describe an optimal control problem with one control variable (n_t) and one state variable (q_t). Under some simplifying assumptions, Dean (1991) shows that in the short term the τ parameter can be approximated by:

$$\tau = (M + h)^{-1} \qquad [5]$$

where M represents the height of the beach berm (in meters above sea level), and h represents the "depth of closure" (in meters below sea level). See Figure 1. The erosion parameter θ reflects average annual beach erosion caused by coastal storms and the background rate of sea level rise. Assuming relatively constant sea level (short term) the erosion control problem may be considered time autonomous, and the beach quality transition equation [2] is constant over time. This setup can be used to evaluate beach erosion management programs in the near term.

Empirical evidence suggests that fixed costs are an important part of the economic costs of beach replenishment, as large amounts of capital equipment (e.g. dredges, pumps, pipes, etc.) are required to produce any appreciable amount of replenishment sand. The existence of fixed costs leads to a rotation-type solution (Smith et al. 2007), with intermittent periods of nourishment followed periods of no activity. To incorporate the rotation pattern, I employ numerical dynamic programming to solve the optimization problem in [1-4].[10] This is most readily accomplished by discretizing the state and control spaces and applying Bellman's backward recursion algorithm. The approach of backward recursion is based on Bellman's Principle of Optimality, which states that an optimal policy must constitute an optimum with regard to the remaining periods regardless of preceding decisions. As such, one can solve the problem by working backwards. Bellman's equation for the beach erosion management problem is:

$$V_J(q_t) = \max_{n_t \geq 0} \{WTP(q_t) - C(n_t) + \eta V_{J-1}(q_{t+1})\}, \qquad [6]$$

where η is the discount factor and J represents the number of periods remaining.

By our empirical estimates, $V(.)$ is differentiable in n. To maximize returns from beach management, we require:

[10] The Mangasarian Sufficiency Theorem requires that both $WTP(q_t) - C(N_t)$ and $-\theta + \tau n_t$ be differentiable and concave in n_t, jointly.

$$-\partial C/\partial n_t + \eta\, \partial V_{J\text{-}1}/\partial q_{t+1} \times \tau \le 0\,,\ n_t \ge 0 \qquad [7]$$

$$(\eta \partial V_{J\text{-}1}/\partial q_{t+1} \times \tau \text{-} \partial C/\partial n_t\,) n_t = 0$$

where the first-order condition holds with equality for an interior solution. By the envelope theorem:

$$\partial V_J/\partial q_t = \partial WTP/\partial q_t + \eta\, \partial V_{J\text{-}1}/\partial q_{t+1} \qquad [8]$$

Combining [7] and [8], we have:

$$\partial V_J/\partial q_t = \partial WTP/\partial q_t + [\partial C/\partial n_t]/\,\tau \qquad [9]$$

for t=0,…,T-1 and J=T-1,…,0. Substituting [9] for $\partial V_J/\partial q_t$ and $\partial V_{J\text{-}1}/\partial q_{t+1}$ in [8] we arrive at:

$$[\partial C/\partial n_t]/\,\tau = \eta\, \partial WTP/\partial q_{t+1} + \eta\, [\partial C/\partial n_{t+1}]/\,\tau \qquad [10]$$

Expression [10] is interpreted as follows: at the optimum, the marginal cost of an additional increment to beach width in period *t* should be equal to the present value of the sum of marginal willingness to pay and the marginal cost of an additional increment to beach width in the subsequent period (*t+1*). The first term on the RHS of [10] reflects the benefits of beach replenishment, which begin to accrue in the next period. The second term on the RHS of [10] reflects the foregone cost of beach replenishment in the subsequent period due to action in the current period. Rearranging [10], we have:

$$[\partial C/\partial n_t]/\,\tau = \eta/(1\text{-}\eta)\, \partial WTP/\partial q_{t+1} \qquad [10']$$

which defines optimal beach replenishment by balancing present marginal cost with the present value of the flow of benefits due to replenishment in perpetuity.

Details on the Study Site and Components of the Model

The optimal control model is applied to Tybee Island, the northernmost barrier island in Georgia. Tybee Island is located about 19 miles east of the city of Savannah, and has a relatively small year-round population of less than 3,000 people (1998 estimate). The population grows to approximately 10,000 between May and September, and can exceed 30,000 on peak days in the summer (USACE 1994). It is a primary recreational destination for Savannah residents, as well as visitors from Atlanta and other population centers. Tybee is situated in a meso-tidal (tide-dominated) region, with tides typically ranging from 2 to 3.5 meters. The beach at Tybee Island is 4666 meters long, and the island is 1207 meters wide, on average. Tybee is a fairly typical southeastern barrier island, consisting of geological formations and subjected to climatological conditions that are somewhat similar to other barriers in the region (Clayton et al. 1992). The geographic setting of Georgia is unique in some respects, however, as the bathymetry and orientation of the coastline partially shelter the island from the full brunt of hurricane and tropical storm forces.

We make use of Landry and Hindsley's (2007) hedonic property model for Tybee Island, which includes beach width as an environmental attribute. The dependent variable is the annual rental rate of the property, calculating using a standard amortization formula and the prevailing rate on a 30-year fixed mortgage in the year of the housing sale. They estimate a linear hedonic price function that is quadratic in beach quality, producing the following benefit function:

$$WTP(q_t) = 11{,}200 + 118.37q_t - 1.95q_t^{\,2}. \qquad [11]$$

This benefit functions are scaled to reflect all properties on Tybee Island (approximately 2,795 (Landry, Keeler, and Kriesel 2003)).[11]

Beach replenishment costs are estimated using historical data on beach fill projects. While our primary analytical focus is on the beach profile (vertical transect of the beach), we model replenishment costs in aggregate. That is, we focus on replenishment sand per unit length (n_t) as a choice variable, but estimate the cost function

[11] As the threat of sea level rise becomes more imminent the threat of erosion loss will become more pronounced, and we might expect that willingness-to-pay for beach quality as a form of protection would increase. Our model does not address this aspect. The benefit functions in [11] are snapshots of preferences under the current expectations of sea level rise. Our intentions are to forecast management decisions based on current preferences in order to provide a conceptual framework for management and to make some predictions about the time horizon of management given what is currently revealed about demand for beach quality in the housing and recreation markets.

as $C(N_t)$, where $N_t = n_t \times l$, l representing length of the beach. We do this in order to avoid making assumptions about returns to scale in beach replenishment along the shoreline. The historical cost data includes 365 observations from the Gulf (Texas, Alabama, Mississippi, and Florida), Southeast (Florida, Georgia, South Carolina, and North Carolina), and Mid-Atlantic States (Virginia, Maryland, and Delaware). The data were obtained from Duke University's *Program for the Study of Developed Shorelines*,[12] and extend back to the early 1960s. Monetary costs were converted to 1998 dollars using the all-industry, producer-price index. The dependent variable is total project cost. Total sand volume (N) and the square of sand volume (N^2) are the chief independent variables of the reduced-form equation.[13] The cost function was estimated by least squares dummy variable (LSDV with state fixed effects) and a time trend.[14]

The parameter estimates are presented in table 1. The LSDV estimates have fairly high explanatory power, and all variables are statistically significant except for the Texas, Mississippi, and Virginia intercept terms. Results suggest that the cost function is increasing and convex. The time trend is positive, indicating that costs have been increasing with time, which could reflect dwindling reserves of high quality beach fill sand in close proximity to the shore. Simplifying the cost functions as functions of sand volume per unit length, we have the following:

$$C_A(n_t) = 749{,}361 + 12598.2n_t + 29.1739n_t^2, \quad [12]$$

$$C_B(n_t,t) = -5{,}603{,}137 + 12598.2n_t + 29.1739n_t^2 + 167{,}171 \times t. \quad [12']$$

[12] PSDS website: http://www.env.duke.edu/psds/index.html
[13] The cost function reflects accounting costs, but not the opportunity cost of capital.
[14] Other forms were estimated, but results were similar. LSDV estimation provided the best fit to the data.

In these specifications, the intercept for the state of Georgia is utilized. The time trend is a count variable, starting at 1 for the year 1961. In [12], the time trend is set to 38 (corresponding with technology and resources in the year 1998) and the influence of the time trend is incorporated in the intercept. Equation [12'] allows the time trend to evolve.

As Tybee Island's beach has been intensively managed over the past 30 years, detailed information on the average beach profile is available. The average berm height (M) is 3.35 meters (USACE 1994); the depth of closure (h) is estimated at approximately 7 meters; the background erosion rate (θ) is 0.67056 meters per year, and the median sediment grain ranges in from 0.16mm to 0.22mm in the nearshore region (Applied Technology and Management, Inc. 2002). Using these parameters, the width of the active profile (W – see figure 1) is about 2,377 meters. For the short-run case of a constant sea level, the transition equation is:

$$q_{t+1} - q_t = - 0.67056 + 0.0966\, n_t. \qquad [2]$$

These components of the model are assembled to solve the short-term beach erosion control problem.

Results

A numerical routine for estimating the value function and optimal beach replenishment policy was adapted following Miranda and Fackler (2002). The program implements value function iteration; backward recursion is used to solve the beach erosion problem by starting in the last period and determining the optimal decision rule and resulting maximum value for each possible state. Working backwards, this procedure is repeated for each previous period. The solution at the first period provides the maximum attainable value, and the entire procedure provides a roadmap of optimal policies for each period conditional on the results of the previous period. $V_J(q_t)$ is the value function, which gives the sum of current and discounted future returns to beach quality following the optimal policy of beach replenishment.

Aggregate willingness-to-pay from *WTP* [11] becomes negative at $q \geq 112$ meters. The state space is defined as $0 \leq q \leq 112$, with 0.1 meter increments giving rise to a 1×1121 vector of states. Since q may only take on multiples of 0.1, the background erosion rate (0.67056) can only be approximated at 0.7 meters per year. Likewise, the τ parameter must be approximated at 0.1. The control space was defined over $0 \leq n \leq 1120$, with 1 cubic-meter increments giving rise to a 1×1121 vector of controls.

The short-term control problem exhibits a constant erosion rate; thus, $q_{t+1} = q_t - 0.7 + 0.1 \times n_t$. According to our field observations taken in the spring of 1998, average initial beach quality (q_0) on Tybee Island survey is 23.5 meters. Consider a finite time horizon of 50 years. Figures 1 and 2 depict the optimal state path and control paths, respectively, for initial beach quality of 23.5 meters and a terminal value given by $WTP(q_T)$. The beach quality that maximizes willingness-to-pay is approximately 30 meters. The optimal beach nourishment policy is a rotation of about 13 years; the optimal control is 136m^3 of sand per meter of beach length in the initial period (bolstering beach width to an average of 36.4 meters), followed by 85m^3/m in period 14 (when beach width has degraded to 28 meters), 84 m^3/m in period 24, and 75 m^3/m in period 36. As indicated in figure 2, beach quality is maintained between approximately 28 meters and 36 meters. The existence of fixed costs gives rise to the intermittent nature of the solution near the optimum state. The autonomous nature of the optimization problem gives rise to a constant rotation (Smith, Slott, and Murray 2007), which is approximated in our empirical application due to discretization and the finite time horizon. The present value of the stream of returns associated with the optimal policy is $46 billion. This number includes the total annual value of all property on Tybee Island, reflected the intercept of [11].

If initial beach quality we 60.8 meters or greater, the optimal policy would be no beach replenishment over the entire time interval. At an initial level of 60.8 meters, annual erosion reduces average beach width to 26.5 meters only after 49 years, and thus beach replenishment is not warranted. Conversely, if initial beach quality were 0 meters, the initial replenishment would consist of 260 m^3 of sand per meter of beach length (giving rise to an average of 25.3 meters of beach width) followed by 118m^3/m in the second period (empirical fixed costs are not high enough to justify placing all the sand on the beach in the initial period), after which the approximate rotation of $78m^3/m$ every 11-13 years until period T-1 holds. The optimal solution is approximately the most rapid approach (MRAP) — only approximate because under some initial conditions it takes two periods to reach the neighborhood of the static optimum. A doubling of fixed costs is enough to converge to MRAP. The MRAP solution holds for all initial beach widths below 28.3 meters. The dominance of the magnitude of the benefits over that of the costs leads to this type of solution. This aspect of the problem also makes the solution insensitive to the discount rate.

The intermittent nature of the solution mirrors the structure of a typical beach replenishment project. Beach replenishment is usually conducted every 5-10 years for chronically eroded beaches. While our preliminary results suggest less frequent operations are optimal, the present model may be missing some portion of benefits (those accruing to recreational users) and likely misses some of the fixed or variable costs (including environmental costs).

Long Run Extensions of the Model

The optimal control problem in [1] - [4] may also be set up to examine the long term problem of shoreline recession due to sea level rise. With rising seas, erosive pressure will be increasing, as the barrier island becomes prone to migrating. This characterization of the problem suggests that the erosion parameter θ will be increasing over time, and we must estimate the path of θ_t. We conjecture that the time path of θ will have two distinct segments: the first segment represents an increasing erosion rate due to sea level rise below the mean height of the barrier island, while the second segment deals with sea level rise above the mean height of the island. Once sea level rise reaches this critical level, the amount of sand required to maintain the barrier island would increase dramatically as sand must be added to raise the entire island. Incorporating the dynamics of sea level rise requires a non-autonomous transition equation:

$$q_{t+1} - q_t = - \theta - \Delta\theta_t + \tau\, n_t = - \theta_t + \tau\, n_t \qquad [2']$$

Dean (1991) and Dean and Maurmeyer (1983) suggest that the non-autonomous portion for a stabilized (non-migrating) barrier island is given by:

$$\Delta\theta_t = (W\Delta S \times t) / (M+h), \qquad [2.1']$$

where W is the width of the active beach profile (the horizontal distance from the mean high-water line to the depth of closure), ΔS is the increase in sea level per unit time, M is height of the beach berm, and h is the depth of closure.

Once sea level rise reaches a critical point, the barrier island will be more prone to landward migration. Under this scenario, the annual erosion rate is augmented by:

$$\Delta\tilde{\theta}_t = (W + w + W_l)\,\Delta S \times t\, /\, [(M + h)\text{-}(M_l + h_l)], \qquad [2.2']$$

in which the new terms are W_l - active profile width on the lagoon side, w - width of the island, M_l - height of the beach berm on the lagoon side, and h_l - depth of closure on the lagoon side. See Figure 2. At this critical point of sea level rise, the τ parameter will change as well:

$$\tilde{\tau} = [(M + h) - (M_l + h_l)]^{-1} \qquad [2.3']$$

To maintain the elevation of the island relative to sea level, the requisite volume of sand per unit shoreline length is approximately $(W + w + W_l)\Delta S$. This would entail quite an engineering feat, and this type of operation is not reflected in any historical beach management data.

Under condition [2'] the coastal erosion problem is non-autonomous; the erosion rate evolves, as does the conversion parameter (after the critical point). The erosion trajectories specified in [2.1'] and [2.2'] are increasing monotonic functions of time. They are intended to represent escalating erosive pressures associated with sea level rise. In the long run, we can consider the selection of terminal time as a management parameter. The time horizon of beach replenishment will depend upon the recreational and protective services of the beach (represented by *WTP*), the rate of sea level rise, and the sensitivity of management costs to sea level rise. If T is free, it should be chosen such that the Hamiltonian expression of the problem in [1] - [4] evaluated at the terminal time is zero (Chiang 1991). Since erosion is increasing monotonically with sea level rise, the replenishment costs of producing a given beach width, conditional on some arbitrary starting point, should be increasing monotonically as well. If economic returns from beach quality are represented by a concave function, the benefits of beach quality are bounded. Under these circumstances, the terminal time is implicitly defined by the balance of benefits and costs. Incorporating a transversality condition and assuming the shadow value of beach quality is driven to zero, the terminal time is implicitly defined by:

$$WTP(q_{T-1}) = C(n_{T-1} \times l) . \qquad [13]$$

Condition [13] indicates that the benefits of beach management must equal the costs in the penultimate period. If the assumption that costs increase monotonically with sea level rise and economic returns from beach quality are bounded, this condition implicitly defines the time at which beach replenishment should be abandoned. In the absence of beach replenishment, a policy of shoreline retreat is implicit. The extension to long-run applications remains a topic for future empirical work.

Conclusions

Beach erosion is a significant problem along America's coastline, and the prospects of sea level rise offer more complications and higher stakes. There is a large amount of property exposed to the risks associated with living on the shore, and management of these risks can have dramatic effects on the beach, a valuable public resource. Using non-market valuation to quantify the returns from beach quality and parameterized cost estimates, we have laid out a conceptual and empirical model of beach erosion management using optimal control theory. The framework focuses on the stream of services produced by barrier beaches, and employs beach replenishment as the management control, with an implicit policy of shoreline retreat inherent in the sustained absence of control. By applying numerical dynamic programming we have shown how this model can be used to determine efficient management of barrier island beaches in the short run. Our model differs from current approaches to coastal erosion management by incorporating both active and passive management regimes and explicitly accounting for the dynamic adjustment process of beaches.

We use Tybee Island, Georgia as a study site and show the optimal short run management strategy for Tybee's beach given our empirical benefit and cost estimates. The short run optimal control problem is time autonomous, which combined with fixed costs of breach replenishment given rise to an intermittent control that buffets the static optimal beach width with period control interventions on the order of every 10 - 13 years. Several caveats apply to these results: i) we focus only on a profile transect for management, and thus neglect the longshore dimension (as well as external benefits and costs that may accrue to adjacent beaches); ii) our benefit measures reflect only willingness to pay of property owners, and thus are limited in scope; iii) the erosion rate is assumed linear, implying that the replenished beach realizes an instantaneous adjustment to equilibrium (unlike Smith, Slott, and Murray 2007); and iv) only engineering and planning costs are considered (neglecting non-market costs of environmental impacts on benthic and beach organisms).

Future research will focus on employing improved benefit estimates to better represent economic returns from beach quality for both homeowners and recreational users, as well as better estimates of the economic cost of beach replenishment (incorporating better measures of opportunity cost of capital, more realistic fixed costs, and environmental impacts). In addition, future work will explore the management problem under sea level rise, specifically estimation of the time horizon of active management under various assumptions about sea level rise and the trajectory of property values. Previous research on coastal protection (Yohe, Neumann, and Ameden 1995) suggests that the primary issue associated with managing barrier islands via beach replenishment under sea level rise is determining when to stop such an operation. That is, at what point should we give up trying to preserve barrier beaches? Our model offers a framework for making a determination of the terminal management time.

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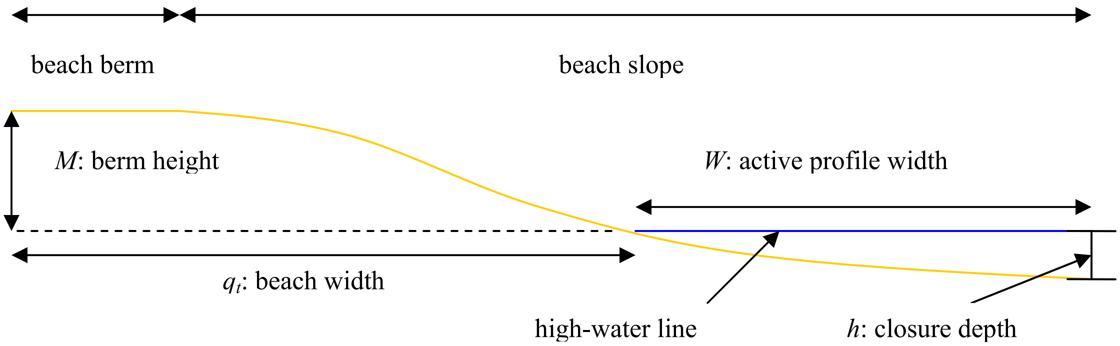

Figure 1: The Orthogonal Beach Profile

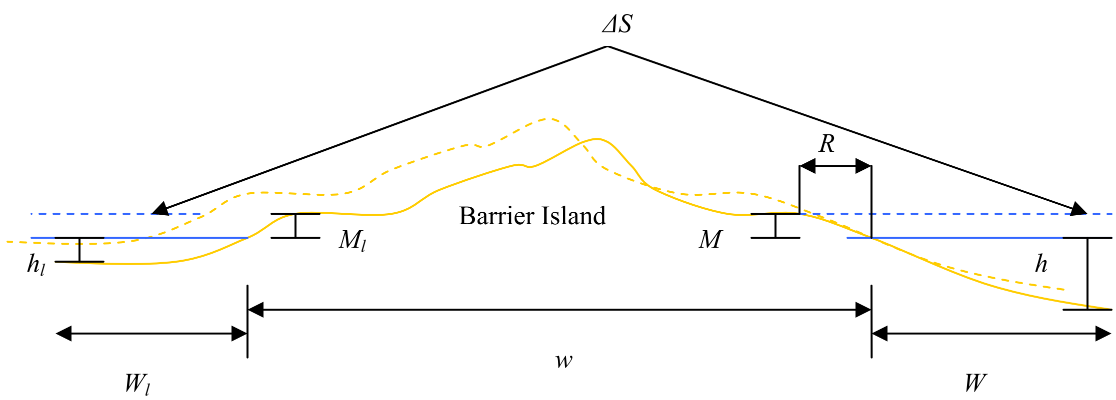

Figure 2: Barrier Island Recession due to Sea Level Rise – Initial forms of island (gold) and sea level (blue) are indicated by solid lines; subsequent forms are indicated by dashed lines; *ΔS* is the change in sea level; *R* is horizontal retreat distant; M_l and *M* are berm heights on the lagoon and ocean side; h_l and *h* are closure depths on the lagoon and ocean side; W_l and *W* are active profile width on the lagoon and ocean side; and *w* is island width. [Adapted from Dean and Maurmeyer 1983]

Table 1: LSDV Estimates of Beach Replenishment Cost Function

Variable	Definition	Coefficient	Standard error
N	sand volume (cubic meters)	2.70152*	1.60590
N2	square of sand volume	0.00000134*	0.00000073
texa	intercept for Texas	-3999311	3333382
loui	intercept for Louisiana	-4691012**	2262796
miss	intercept for Mississippi	-5288455	5754290
bama	intercept for Alabama	-5531081*	2982792
flor	intercept for Florida	-3542236***	797561
geor	intercept for Georgia	-5603137**	2674120
scar	intercept for South Carolina	-4803018***	1486330
ncar	intercept for North Carolina	-5141801***	1116034
virg	intercept for Virginia	1917744	1369406
mary	intercept for Maryland	14055147***	2642486
dela	intercept for Delaware	-3103231***	1056543
time	time trend (1961=1; 2002=42)	167171***	27904
Dependent Variable=cost in 1998$; # obs.=365; R^2=0.4137; F_{stat} (df=14) = 17.64; *=statistically significant at α=10%; **=statistically significant at α=5%; ***=statistically significant at α=1%			

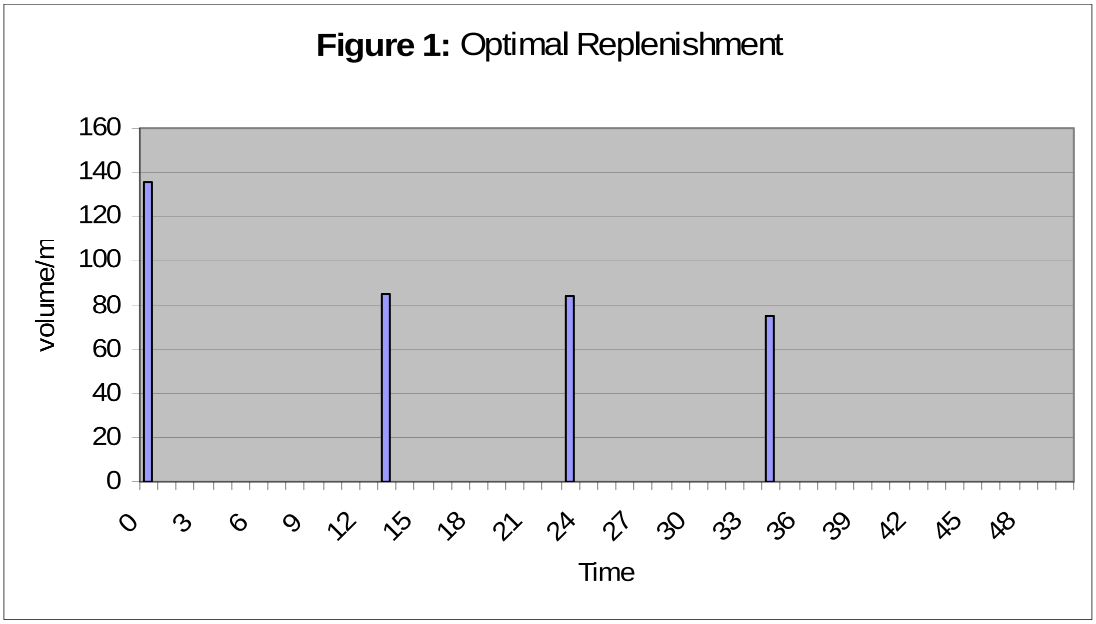
Figure 1: Optimal Replenishment
volume/m
160
140
120
100
80
60
40
20
0
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48
Time

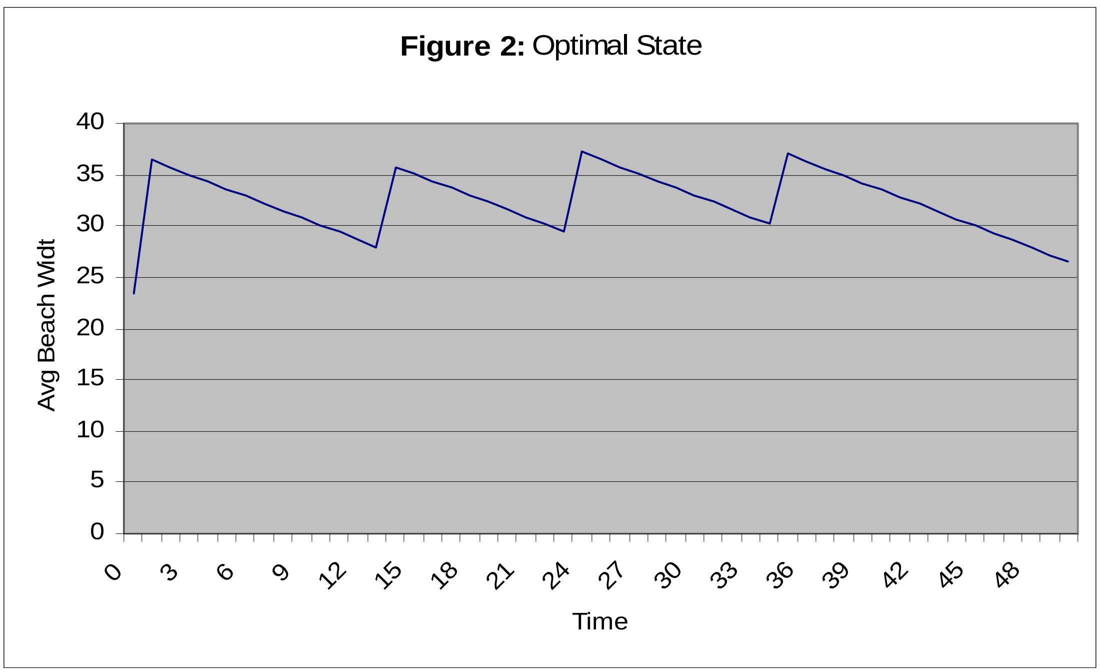
Figure 2: Optimal State
Avg Beach Widt
40
35
30
25
20
15
10
5
0
0 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48
Time