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Multiple Reserve Requirements, Exchange Rates, Sudden Stops and Equilibrium Dynamics in a Small Open Economy[♦]

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Abstract

We model a typical Asian-crisis-economy using dynamic general equilibrium techniques. Meaningful exchange rates obtain from nontrivial demands for fiat currencies. Sudden stops/bank-panics are possible, and key for evaluating the relative merits of alternative exchange rate regimes in promoting stability. Strategic complementarities contribute to the severe indeterminacy of the continuum of equilibria; there is a strong association between the scope for existence and indeterminacy of equilibria, the properties along dynamic paths and the underlying policy regime. Binding multiple reserve requirements reduce the scope for financial fragility and panic equilibria; backing the money supply acts as a stabilizer only in fixed regimes.

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Keywords: Sudden stops; Exchange rate regimes; Multiple reserve requirements.

1. Introduction

We study the interaction between monetary policies and alternative exchange rate regimes to ascertain the probability of a crisis, building from the characteristics of the Asian-crisis countries in 1997. Our broader goal is to reinforce and fill in the link between the overexpansion of the financial system, banking crises, and exchange rate regimes/monetary policy that we find lacking in the literature. With this in mind, we build a Dynamic Stochastic General Equilibrium Model (DSGE) --from micro-foundations-- replicating a small, open economy (SOE) with a nontrivial banking system, such as one of the 1997 East Asian countries. Two words of caution to the reader: First, this paper does not aim, from a historical point of view, to show the success of a particular monetary policy in place either in defending the national currency or in managing contagion at the time of the crisis. Our goal, instead, takes the form of a "what if:" what if a typical Asian-crisis-country were

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to implement a policy of multiple reserve requirements with backing of the domestic money supply, and how would it work under alternative exchange rate arrangements? Thus, we look forward and aim to suggest policy options that may help these countries maintain stability in case a similar crisis was to hit again. Second, at this time, we do not consider economic activity explicitly other than in the financial sector.

Our model captures all five stylized facts of the East Asian countries at the time of the crisis. It is general knowledge that Indonesia, South Korea, and Thailand were the countries most affected by the East Asian 1997/98 crisis, followed by Malaysia, Laos and the Philippines. There are five stylized facts shared by these countries at the time of the crisis that we want to emphasize. 1) Increased risky-lending behavior by banks led to a boom in private borrowing financed by non-performing loans[1]. 2) The lack of sound financial structure worsened with the ill-oriented process of financial and capital liberalization[2]. 3) Banks' financial assets constituted the majority of their total assets –instead, for instance, of financing in capital markets. 4) Borrowing from foreign banks was a significant portion of domestic banks' loans. 5) The majority of these countries had intermediate pegs in place. According to the standard chronology of the crisis, the floating of the baht in July 1997 in Thailand triggered the crisis. A subsequent change in expectations led to the depreciation of most currencies in the region, bank runs, rapid withdrawals of foreign capital --a *sudden stop*—and a dramatic economic downturn followed. Unlike previous crises originated from fiscal imbalances and/or trade deficits, the Asian crises shed light on the increased risky behavior and the overexpansion of the banking system.

To build the framework that we just described, we used three building blocks that took us closer to our goal systematically. In the first block, we model explicitly the behavior of individuals and obtain the micro-foundations for our general equilibrium model. In the second block, we introduce alternative exchange rate regimes with their associated monetary policy rules. It is a well-established fact that for economies open to international capital flows, the choice of exchange rate regime is central to explain the vulnerability and fragility of financial markets, as well as domestic price stability and long-run viability. Tables 1.A and 1.B summarize the exchange rate arrangements in the Asian countries. During most of the 1980s and the first part of the 1990s, Indonesia, Korea, Thailand and Malaysia had managed floating arrangements --an intermediate peg--, while Philippines had free floating. However, there were some important differences after the 1997 crises: Philippines continued with free floating, Indonesia, Korea and Thailand moved from intermediate pegs to free floating as well, but Malaysia had a very hard peg in place. These facts make our comparison of the relative merits of the two sets of policy rules relevant in the presence of binding multiple reserve requirements.

The third building block may allow one to infer behavior from a particular set of circumstances: we may be able to separate and identify causes and

[1] One may also think of this fact in the context of the subprime mortgage crisis that started in the second-part of 2007 in the U.S.
[2] See Lindgren et al (1999) and Kishi and Okuda (2001).

consequences by studying separately and jointly the main stylized facts of sudden stops and bank-runs in similar economies[3].

We consider two potential causes of crises: a crisis comes to our model either in the form of a sudden stop of foreign credit (*intrinsic* uncertainty) or in the form of a panic among national depositors (*extrinsic* uncertainty.) We put most of our effort on the distinguishing characteristic of the former but do not neglect the fact that a self-fulfilling panic and run may implicitly aggravate a crisis.

We believe that we improve on C-V in at least three dimensions. First, C-V attached intrinsic value to currencies they intended to be fiat[4]. We instead take multiple fiat currencies –domestic and foreign—a bit more seriously, and introduce non-trivial demands for them. In particular, banks must hold a fraction of their deposits as unremunerated currency reserves: a fraction to be held in the form of domestic currency and another fraction in the form of foreign currency. Then, fiat money instead enters our model by the regulation that governs the multiple reserve requirements in this economy[5], the implications being that: 1) there is a meaningful nominal exchange rate in our model, and 2) this nominal exchange rate will be determined according to the exchange rate regime and the monetary policy in place. In second place, we use a DSGE model in an economy with an infinite horizon, as is the OG. Thus, we are able to discuss the interesting equilibrium dynamics defining each exchange rate arrangement, as opposed to both D-D and C-V. In third place, we improve the way in which we introduce and treat potential crises, The potential for strategic complementarities and the realization of self-fulfilling prophesies is ever present in alternative versions of the OG model with outside assets in general, and models with one or more fiat currencies in particular[6], and, of course, in our model. In such contexts, the presence of informational and institutional frictions can exacerbate situations that are already problematic, such as credit rationing, financial repression and endogenously arising volatility, thus complicating the standard analysis of separating and pooling equilibria. Thus, the appropriate utilization of the information and action sets available to agents at all points in time is critical. In this respect, we reformulate the sequential checking constraint by depositors and devise a re-optimization problem by banks after a sudden stop[7].

Our results show the existence of a continuum of equilibria that are indeterminate in two ways: 1) an allocation may be consistent with a continuum of relative price vectors, and 2) a vector of relative prices may

[3] See Kaminsky (2003) for details.
[4] In their model, people held domestic currency because they derived utility from it, which in itself attached intrinsic value to the currency they intended to be fiat.
[5] See Hernandez-Verme (2004) for the original discussion.
[6] The recent literature on open economy macroeconomics has used intensively self-fulfilling prophecies as a tool that may lead to very important underlying explanations for financial fragility, currency crises and/or speculative attacks. See Cole and Kehoe (1996) and Obstfeld (1996).
[7] In particular, one argument of the C-V framework was that when the probability of a crisis is public information, each agent in this economy must use this information when contemplating optimal plans of action at the beginning of every date, and, as result, the optimal behavior of agents is invariant with respect to whether the crisis was realized or not. Alternatively, we introduce the potential for uncertainty of the crisis by using a sunspot variable: a random variable unconnected to the fundamentals of the economy and that expresses the extrinsic uncertainty.

be consistent with several different allocations. There is a strong association between the scope for existence and indeterminacy of equilibria, the properties along dynamic paths and the underlying policy regime. Binding multiple reserve requirements may help in reducing the scope for financial fragility and panic equilibria, but backing of the domestic money supply has stabilizing effects only under fixed exchange rates.

The remainder of the paper proceeds as follows. In Sections 2 and 3, we analyze the properties of stationary and dynamic equilibria under the alternative exchange rate regimes where no crises are possible in equilibrium. In Section 4, we allow for the possibility of crises by introducing extrinsic and intrinsic uncertainties. Section 5 concludes.

2. Floating Exchange Rates: the Case of Indonesia, Korea, Philippines and Thailand

In this section, we build the model of a SOE that captures the main stylized characteristics shared by the Indonesian, Korean, Filipino and Thai economies at the time of the crisis. Here, we focus on the construction of the general equilibrium and, thus, we do not allow for any event that could lead to a crisis of any type. The reader interested can find the analysis of crises in Section 4.

The (private) banking sector is a net debtor with respect to the rest of the world, and there is an exogenous and binding upper limit to foreign credit faced by domestic banks at each point in time, so that credit is always rationed. We will observe *ex-ante* identical domestic agents who face uncertainty as to their preferences types. The distribution of this shock is public information, but its realization is known only by the private agents. Our model has the potential for strategic complementarities, taking the form of a standard problem in coordination that may lead to crises of a self-fulfilling type[8]. We will see that two fiat national currencies can potentially circulate simultaneously: a domestic fiat currency and a foreign fiat currency. The legal regulations in financial intermediation and foreign exchange establish the following: 1) All intermediated domestic investment is subject to multiple, unremunerated and binding reserve requirements. 2) A flexible exchange rate regime is in place, and thus the nominal exchange rate will be market-determined; and 3) There are no legal domestic restrictions on either using foreign currency or on obtaining foreign credit.

2.1 The Environment

Consider a *pure exchange*, SOE consisting of an infinite sequence of two-date-lived, overlapping generations. Time is discrete, and indexed by $t = 1, 2, 3, \ldots$. Standard analysis of an overlapping-generations economy typically groups households into two categories: all the future generations versus the generation of initial old. Moreover, we will observe four groups of players in this model economy: households, domestic banks, foreign

[8] The decisions made by individual agents will be intertwined with the choices of other agents, giving rise to strategic interdependence between an agent's actions and the actions of others.

banks and the domestic monetary authority. Foreign banks will lend to domestic banks inelastically at the world interest rates and up to the exogenous binding limit. The monetary authority in this model economy is in charge of choosing the combination of monetary policies consistent with floating exchange rates.

Each of the future generations, on the one hand, consists of a continuum of households with unit mass. A household born at date t is young at date t and old at date $t+1$. Households within a generation are *ex ante* identical, but they can become of one of the following types before the end of their youth: impatient, with probability $\lambda \in (0,1)$, and patient, with probability $(1-\lambda)$. As we will see later on, impatient households will derive utility from consuming before the end of their youth ($c_{1,t}$,) while patient households will derive utility only from consuming in their old age ($c_{2,t+1}$.) On the other hand, at $t=0$, there is a generation of initial old. The initial old consist of a continuum of old households with unit mass. The fraction $(1-\lambda)$ of these initial old are of the patient type.

Each date has two parts that we will call morning and afternoon, since different types of interaction will take place in each of them[9]. Domestic banks will turn out to be coalitions of individual households, they will be competitive, and we can assume then that they are identical. The latter facilitates the analysis by allowing the examination of only a representative bank.

Each date, there is a single endowment, tradable good. This good is homogeneous across households and countries, but it cannot be produced anywhere. When young, a domestic household receives w units of the single good, first thing in the morning. Old agents receive no endowments of any type.

The following equation represents the expected lifetime utility of an individual born at date t, with the information available at the beginning of this date.

$$E_t\left[u\left(c_{1,t},c_{2,t+1}\right)\right] = \lambda \cdot \ln c_{1,t} + \left(1-\lambda\right) \cdot \ln c_{2,t+1}. \qquad (1)$$

Households and domestic banks have access to the following storage/investment technology: for one unit of good invested –not consumed-- at the beginning of date t, the household receives the return $R>1$ goods at the end of date $t+1$. However, she would receive only a return of $r<1$ if she were to liquidate the investment early, by the end of date t [10]. Then, the condition to promote truth-telling can be written as

$$c_{2,t+1} \geq r \cdot c_{1,t}. \qquad (2)$$

The inequality above must hold as the incentive-compatibility or self-selection condition that allows no motivation for patient agents to misrepresent their types. In the remainder of this section and in Section 3, we assume that the inequality (2) holds while we build the general equilibrium.

[9] This separation is indispensable to our setup, because the sequence in which events take place and information becomes available are essential in determining the coordination between agents (or the lack of) and the nature of the information problem in place.

[10] The only available technology for the short-term investment is this same storage technology. For instance, if an agent liquidates early, she would get the return $r<1$ per good at the end of date t, and if she stores the proceeds again between the end of date t and the end of date $t+1$, then she will get the return $r \cdot r = r^2 < 1 < R$ per good at the end of date $t+1$. Thus, the long-term storage technology dominates early liquidation in rates of return.

Multiple Fiat Currencies. Two fiat and outside national currencies may circulate in the small, open economy at any point in time. The first one is the domestic fiat national currency, while the second is a foreign currency that serves also the purpose of international currency. To fix ideas, we will call the domestic currency the *won*[11] and the foreign/international currency the US dollar [12].

On the first hand, it is apparent that the monetary authority in the domestic country has the monopoly in issuing *wons*, where M_t is the outstanding nominal stock of *wons* at the end of date t. The domestic price level p_t represents the number of *wons* to be exchanged for one unit of the single good at date t, and $\left(p_t / p_{t+1}\right) > 0$ is the real return on *wons*. As is standard in economies with floating exchange rates, the monetary authority has the control over the nominal aggregate supply of *wons*, as we will see in detail in section 2.3. On the other hand, the US dollar may circulate in the domestic economy together with the *won*, and Q_t represents the outstanding stock of foreign currency in the domestic country at the end of date t. The exogenous world price level p_t^* represents the number of US dollars that households need to exchange for one unit of the single good at date t, while $\left(p_t^* / p_{t+1}^*\right) = \left(1+\sigma^*\right)^{-1} > 0$ represents the constant real return on US dollars, where $\sigma^* > -1$ is the exogenous net inflation rate in the rest of the world. It is apparent that Q_t is endogenous, and it will depend on a group of variables such as foreign credit constraints binding, relative prices and policy rules, among others. Finally, we use e_t to denote the market-determined nominal exchange rate, measured as the number of *wons* exchanged for one US dollar.

We also assume that there is free international capital mobility, free international trade with homogeneous goods and no legal restrictions to the use of foreign currency in the domestic country. As a result, the Law of One Price will hold in equilibrium, so that $e_t \cdot p_t^* = p_t$ is always satisfied.

With respect to the general guidelines for monetary policy, the monetary authority in the domestic economy accomplishes all injections and/or withdrawals of *wons* through lump-sum transfers. In particular, each young household will receive the equivalent in *wons* of τ_t goods *ex ante*, at the beginning of date *t*, regardless of type. We have two reasons behind our choice of this particular scheme of monetary transfers: first, it enters the self-selection constraint neutrally without adding to the information problem, and second, it does not require the monetary authority to have additional information that could be of a private nature[13].

At $t = 0$, the initial-old generation behaves as any old agent from the future generations would. In particular, a fraction $\left(1-\lambda\right)$ of these initial old individuals would be of the patient type, and, thus, would like to consume now. In standard models with overlapping generations, the initial conditions of the economy

[11] The won is the national currency of the Republic of Korea.
[12] Obviously, in the case of the sub-prime mortgage crisis, the domestic currency would be the U.S. dollar.
[13] We must point out that, in this framework, the question of "who gets what and when" is not a trivial one, and different answers to it may have a crucial influence on the equilibrium results. The transfer scheme we propose may not be neutral, in the sense that it may affect the self-selection constraint, but it does affect both sides of it. In Appendix 1, we present results for the alternative case where the monetary authority gives transfers only to young agents who claim to be of the impatient type, after the realization of types.

describe the initial stock of the different assets that exist in the economy, and they play a key role. Typically, the initial stocks of outside assets indicate the "endowments" of the members of the generation of initial old individuals .In the present case, there are two initial conditions: $M_0 > 0$ and $Q_0 > 0$ are given, and they are distributed equally among the patient initial old such that each has the equivalent of $\left(M_0 + e_0 \cdot Q_0\right)/\left(1-\lambda\right) \cdot p_0$ goods early in date $t = 0$.

2.2 Financial Intermediation

In our model, foreign banks play the somehow impersonal role of suppliers of international liquidity through foreign-credit instruments that may take different forms. Thus, the private banking sector in equilibrium is a net debtor of the rest of the world.

Domestic banks follow the standard D-D set up: a representative bank arises endogenously and pools together the resources owned by households with the purpose of providing them with partial insurance against the uncertainty of their potential types and with feasible allocations that are Pareto superior to autarky[14]. In the context of a small, open economy, domestic households may also benefit from pooling their resources and acting together whenever such economy does not display aggregate uncertainty, as is the case in our model economy.

Following the standard practice in the literature, a representative bank that starts business at date t will behave so that it maximizes the expected utility of the individual households born at date t, given by equation (1), which is also in the bank's best interest as well. Henceforth, when we use the term "banks" without further qualification, we refer to domestic banks starting business at t.

Access to Foreign Credit Markets. Only banks may access the world credit markets by trading with foreign banks in several primary debt markets, the idea being that the debt-instruments available may provide them with liquidity in a variety of terms and/or dates of maturity.

Banks have access to three different foreign-debt instruments. The amounts traded of the different instruments form the debt-structure vector denoted by $\left(d_{0,t}, d_{1,t+1}, d_{2,t+1}\right)$, where all amounts are expressed in terms of the single good[15]. The vector of relevant prices associated with this debt-structure is denoted by $\left(r_0^*, r_1^*, r_2^*\right) \gg 1$. $d_{0,t}$ is a short-term *intra-date* loan issued early-morning in date t and maturing in late-afternoon of the same date, while $d_{1,t+1}$ stands for a short-term *inter-date* loan issued in the late afternoon of date t and maturing late in the afternoon of date $t+1$. Finally, $d_{2,t+1}$ is a *long-term* loan issued early in the morning of date t and maturing late in the afternoon of date $t+1$. We assume that each of the elements in the price vector $\left(r_0^*, r_1^*, r_2^*\right) \gg 1$ is a time-invariant and exogenous gross real interest rate determined in the

[14] See the section on deposit contracts for more details. Notice also that the intermediation equilibrium is not Pareto optimal since early liquidation is costly.
[15] This treatment is standard in the literature. For the reader interested in the analysis of nominal debt denominated in different currencies, see Freeman and Tabellini (1998), Chang ( ) and Hernandez-Verme (2008.)

appropriate world financial market such that $\left(d_{0,t}, d_{1,t+1}, d_{2,t+1}\right) > 0$ and the vector $(0,0,0)$ never obtains in equilibrium. Obviously, it follows that banks are net debtors of the rest of the world.

Banks also face exogenous borrowing constraints on standing debt at each date, represented by

$$d_{0,t} + d_{2,t+1} \leq f_0 , \tag{3a}$$

$$d_{1,t+1} + d_{2,t+1} \leq f_1 . \tag{3b}$$

$f_0 > 0$ and $f_1 > 0$ are time-invariant and measured in terms of the single good. They are chosen exogenously by foreign banks, and represent the maximum foreign credit available at dates t and $t+1$, respectively. (3a) and (3b) limit the total amount of foreign debt standing at each date, and we restrict our attention to allocations where (3a) and (3b) bind, so that foreign credit is rationed.

Fractional-Reserves Banking and Multiple Reserve Requirements. Multiple and unremunerated reserve requirements in our model follow Hernandez-Verme (2004.) All investment done by banks is subject to the financial regulations of the domestic country[16]. Out of the total deposits, a fraction must be held as currency reserves and only the remainder can be invested. A part of the currency reserves must be held in the form of *wons*, while the rest must be denominated in US dollars. In particular, the policy parameter $\phi_d \in (0,1)$ denotes the fraction of total deposits that the banks must hold as currency reserves in the form of *wons*. When held between dates t and $t+1$, domestic currency reserves earn the same return as real *wons* balances, namely $\left(p_t / p_{t+1}\right)$. Similarly, $\phi_f \in (0,1)$ denotes the fraction of deposits that banks must hold in the form of foreign currency, with the return $\left(p_t^* / p_{t+1}^*\right)$. Obviously, $\phi_d + \phi_f < 1$ must hold, where $\left(1 - \phi_d - \phi_f\right) > 0$ stands for the fraction of total deposits that banks can invest long-term. Finally, we must mention that we will focus on allocations where both reserves requirements are binding. This will transpire when $\left(p_t / p_{t+1}\right) < R$ and $\left(p_t^* / p_{t+1}^*\right) < R$ hold.

Timing of Transactions[17]**.** In our model, there are no transactions among individual agents of any age or type, either domestically or with the rest of the world. All transactions take place through the banks. Thus, banks in this model are inherently financial intermediaries.

Individuals born at t live for four sub-dates: the morning and afternoon of date t, and the morning and afternoon of date $t+1$. We now proceed to describe the transactions that take place each sub-date. Notice that only in this section we setup the budget constraints with the amount of early liquidation l_t to facilitate a full understanding of the general bank's problem, but in what follows we will return to the case where $l_t = 0$

The morning of date t ***:***. Young individuals born first thing in the morning of date t have two sources of funds at this point: their endowment of w units of the single good, and the transfer of τ_t goods from the monetary authority. Each of these young individuals deposits $w + \tau_t$ goods in a bank. On the other side of the

[16] We further assume that all domestic investment is intermediated.
[17] Timing of transaction in figure is available in Appendix.

financial market, we have banks receiving these deposits. Banks have also the ability to borrow from the rest of the world the amount of $d_{0,t} + d_{2,t+1}$ goods. They must also set aside the required currency reserves and bring them to their reserves account in the monetary authority. Banks combine these resources in order to finance the long-term investment k_t that uses the domestic storage technology, leading to the budget constraint

$$k_t \leq d_{0,t} + d_{2,t+1} + (1-\phi_f - \phi_d)\cdot(w+\tau_t)\,. \tag{4}$$

The afternoon of date t***:*** Individual agents learn their types in the early afternoon of time *t*. Impatient agents will withdraw $c_{1,t}$ goods and consume them, while a patient agent would not withdraw, provided (2) holds. Banks need to pay withdrawals in the amount of $\lambda \cdot c_{1,t}$, and to repay $r_0^* \cdot d_{0t}$ of principal plus interest of intra-date foreign debt[18]. They also have a potential source of new funds at this point in the new short-tem foreign loan of $d_{1,t+1}$ goods. In case more funds were required, banks could liquidate early a part of the long-term investment (l_t goods,) but they try to avoid doing so, since early liquidation is costly[19]. Summarizing, the budget constraint faced by a bank at the end of date *t* is

$$\lambda \cdot c_{1,t} + r_0^* \cdot d_{0,t} \leq r \cdot l_t + d_{1,t+1}\,. \tag{5}$$

The morning of date $t+1$***:*** There are no "active" transactions in this sub-date. Both patient agents and banks wait for the duration of this sub-date. If all agents behave according to their true type, then all impatient agents have already consumed in the morning of date t.

The afternoon of date $t+1$***:*** At the end of date $t+1$, agents of the patient type who behave according to their true type wish to withdraw funds to consume in their old-age ($c_{2,t+1}$ goods each.) Repayments of the long-term foreign debt $\left(r_2^* \cdot d_{2,t+1}\right)$ and the new short-term foreign debt $\left(r_1^* \cdot d_{1,t+1}\right)$ are due as well. A bank still in operation will use the return of the remaining long-term investment -given by $R\cdot\left(k_t - l_t\right)$ goods- together with the gross real return on its currency reserves to pay its obligations. It is helpful to notice at this point that one of the consequences of the regulations on reserve requirements concerning the financial sector in this economy is that banks have an additional source of funds at the end of $t+1$ –even if they yield each a dominated real rate of return. This contributes to reduce the likelihood of a panic of patient depositors who did not want to withdraw at the end of date *t*. In brief, the budget constraint faced by a bank in late-afternoon of date $t+1$ is given by

$$\left(1-\lambda\right)\cdot c_{2t+1} + r_2^* \cdot d_{2,t+1} + r_1^* \cdot d_{1,t+1} \leq R\cdot(k_t - l_t) + \phi_d \cdot \left(\frac{p_t}{p_{t+1}}\right)\cdot\left(w+\tau_t\right) + \frac{\phi_f \cdot \left(w+\tau_t\right)}{\left(1+\sigma^*\right)}\,. \tag{6}$$

[18] The parameter λ is public information, but all the banks know is that there are λ individuals of the impatient type and each shall withdraw $c_{1,t}$ goods. Banks do not know the identities of the truly impatient agents, and so they would allow total withdrawals amounting to $\lambda \cdot c_{1,t}$ goods to agents pretending to be of the impatient type, following a sequential service constraint of the form "first come, first served." In case the bank exhausts its resources before repaying all foreign loans that are due and all depositors left in the line for withdrawals, the bank closes, and any future payments contracted by the bank are lost.

[19] In the sense that it obtains the effective return $r < 1 < R$ instead. Moreover, we could think of short-term inter-date debt and early liquidations as substitute sources of liquidity for banks.

Deposit Contracts. Domestic individual agents face uncertainty about the type they will become at the end of their youth. Moreover, once realized, this information is private to each individual agent. Under these circumstances, the representative bank aims to protect itself by using some kind of self-selection mechanism. Such a mechanism is designed to give individual agents the right incentives to behave according to their true type. In our particular case, this mechanism takes the form of the truth-telling constraint given by the inequality in (2). The severity of this private information problem in a particular state of the world will determine whether (2) will bind or not. For our convenience, henceforth, all of our analysis will refer to the general case of a generation born at date *t*, unless we explicitly say otherwise.

Autarkic Equilibrium. In the absence of financial intermediation, individuals cannot benefit from pooling their resources and there are no insurance schemes available to them. Thus, they could save only through their investment/storage technology. When the information about types is realized at the end of their youth, individual agents in autarky face a feasible set in the space of state-contingent commodities $\left(c_{1,t}, c_{2t+1}\right)$ that contains only the *point* $\left(r \cdot \left(w + \tau_t\right), R \cdot \left(w + \tau_t\right)\right)$.

Financial Intermediation. Recall that representative banks are coalitions of individuals in our model economy. These banks will offer to individuals a deposit contract consisting of the state-contingent pair $\left(c_{1,t}, c_{2,t+1}\right)$. Banks design this contract by choosing the pair $\left(c_{1,t}, c_{2,t+1}\right)$ that maximizes the individuals' lifetime utility described in (1), subject to the constraints (2)-(6). Interestingly, when financial intermediation is available to individuals, the feasible set consists of a continuum of state-contingent commodities. These deposit contracts may provide individuals with allocations that are Pareto-superior to that of autarky. In particular, notice that banks offer partial insurance against the uncertainty of types in the deposit contract, and they are capable of doing so due to their ability to pool the individuals' resources. Individuals, in turn, are willing to sacrifice a little of potential returns in exchange for this insurance, so that $r \cdot \left(w + \tau_t\right) < c_{1,t} < c_{2,t+1} < R \cdot \left(w + \tau_t\right)$ holds.

2.3 Conducting Monetary Policy under Floating Exchange Rates

In this model economy with flexible exchange rates, the monetary authority prioritizes two important aspects when conducting monetary policy: 1) the setting of the rate of domestic money growth, and 2) the choice of the appropriate backing of the domestic money supply with foreign-reserve assets[20].

In the case of the first aspect, the growth rate of the domestic money supply becomes a tool of monetary policy of the utmost importance, as opposed to situations where the money supply is subordinated to other primarily chosen policy objectives. Many reasons may explain the special interest on this policy tool,

[20] In this paper, we do not concern ourselves with the choice of the optimal rate of money growth or the optimal fraction of the money supply to be backed by the monetary authority. We do, however, provide some useful guidelines by discussing their effects on the existence and other properties of steady-state equilibria as well as the properties of the system along dynamic equilibrium paths.

other than the obvious --controlling the evolution of the domestic money supply--. First, it contributes to the control, up to some degree, over the determination of inflation rates, especially --but not only-- in the long-run. Second, it is a mean of giving or taking away incentives to the actors involved with real activity –in our case, financial activity. Third, it can influence the formation of public expectations by keeping reasonable stable rates of inflation as well as the value of the national currency. We adopt the simplest scenario for our model economy, which contemplates the choice and setting, once-and-for-all, of a constant rate of money growth. Such a policy sets the evolution of the supply of *wons* by the rule

$$M_{t+1} = (1+\sigma)\cdot M_t, \forall t > 0, M_0 > 0, \tag{7}$$

where $\sigma > -1$ is the exogenous and constant rate of domestic money growth, chosen by the domestic monetary authority. The latter injects/withdraws money through lump-sum transfers to all young agents in the amount of τ_t goods each.

With respect to the second aspect, backing the domestic money supply is by itself a precautionary mechanism aimed to protecting this economy against potential reversals in the World financial market. This is also of key importance, given the significant amount of action this country has in foreign credit markets. Typically, the monetary authority chooses and sets a fixed fraction of the money supply to be backed or guaranteed. Then, the backing takes the form of holdings of foreign, interest-bearing reserve assets in an amount equal to the dollar-value of this chosen fraction of the supply. This, of course, aims to stabilize the perceived value of money and the willingness of the public to hold it. Thus, the monetary authority holds B_t^* dollars of foreign-reserve assets. These assets yield the interest rate $\tilde{r} \in (1, R)$ every date. These reserve-holdings evolve over time according to the rule

$$B_t^* = \theta\cdot\left(\frac{M_t}{e_t}\right), \tag{8}$$

where $\theta \in [0,1]$ is the exogenous and constant fraction of the *dollar-value* of the supply of *wons* backed by the monetary authority. This policy is a variation of the one used in Hernandez-Verme (2004)[21].

We now define $z_t \equiv (M_t / p_t)$ to be the per capita real balances of *wons* and $b_t^* \equiv (B_t^* / p_t^*)$ to be the per capita holdings of foreign-reserve assets. When the two policy rules in (7) and (8) are adopted and combined by the monetary authority, its budget constraint at each date takes the following form:

$$\tau_t = \frac{M_t - M_{t-1}}{p_t} - \frac{B_t^* - \tilde{r}\cdot\left(p_t^*/p_{t-1}^*\right)\cdot B_{t-1}^*}{p_t^*} = \left(\frac{\sigma}{1+\sigma}\right)\cdot z_t - \left(b_t^* - \tilde{r}\cdot b_{t-1}^*\right) = \left[\left(\frac{\sigma}{1+\sigma}\right) - \theta\right]\cdot z_t + \tilde{r}\cdot\theta\cdot z_{t-1}. \tag{9}$$

Notice that equation (9) is a linear first order difference equation that describes the evolution of the real monetary transfer (τ_t ,) and that its dynamic behavior is inherited from z_t .

[21] Here, we assume for simplicity that there is no backing of domestic deposits. Notice that we use the symbol ξ here to denote the coefficients in the first order difference equation that governs the long-term investment.

2.4 General Equilibrium under Floating Exchange Rates

Several sets of conditions must be satisfied simultaneously. We list them below.

First, there are two conditions on international transactions that apply to our small open economy. One, since the single good is homogeneous across countries and there are no restrictions on the international trade of goods, it follows that purchasing power parity must hold, and, moreover, that the market-determined exchange rate e_t adjusts to ensure this condition is satisfied. Thus, we have

$$e_t \cdot p_t^* = p_t . \qquad (10a)$$

Also, we confine our attention to vectors of prices $\left(r_0^*, r_1^*, r_2^*\right) \gg 1$ that satisfy the following no arbitrage condition, which controls for the different maturities of the three foreign-debt instruments:

$$R = r_2^* = r_0^* \cdot r_1^* . \qquad (10b)$$

Second, there are two constrains on the rates of return in this economy that imply that the reserve requirements are binding. This transpires when the following two conditions hold:

$$R > \left(p_t / p_{t+1}\right) \qquad (11a)$$

$$R > \left(p_t^* / p_{t+1}^*\right) = \left(1+\sigma^*\right)^{-1} . \qquad (11b)$$

Third, we have all the conditions associated with the market for *wons*. One, the domestic price level p_t clears the market for domestic real money balances:

$$z_t = \phi_d \left(w + \tau_t\right) . \qquad (12a)$$

The latter, in turn, leads to the equilibrium return of domestic real money balances

$$p_t / p_{t+1} = \left(1+\sigma\right)^{-1} \left(z_{t+1} / z_t\right) \qquad (12b)$$

and, –using also (9) in the process, to the equilibrium laws of motion of z_t and τ_t, respectively

$$z_t = \alpha_1\left(\sigma\right) + \alpha_2\left(\sigma\right) \cdot z_{t-1} , \qquad (12c)$$

$$\tau_t = \beta_1\left(\sigma\right) + \beta_2\left(\sigma\right) \cdot z_{t-1} , \qquad (12d)$$

where $\mathfrak{M}(\sigma) \equiv \{1 + \theta \cdot \phi_d + \sigma \cdot [1 - \phi_d \cdot (1-\theta)]\}$, $\alpha_1(\sigma) \equiv [\phi_d \cdot w \cdot (1+\sigma)] / \mathfrak{M}(\sigma)$, $\alpha_2(\sigma) \equiv [\theta \cdot \phi_d \cdot \tilde{r} \cdot (1+\sigma)] / \mathfrak{M}(\sigma)$, $\beta_1(\sigma) \equiv [\alpha_1(\sigma) - \phi_d \cdot w] / \phi_d$ and $\beta_2(\sigma) \equiv [\alpha_2(\sigma)] / \phi_d$.

Fourth, there is the market for foreign currency. This market clears when

$$q_t \equiv \left(e_t \cdot Q_t / p_t\right) = \phi_f \cdot \left(w + \tau_t\right) = \phi_f \cdot z_t / \phi_d . \qquad (13a)$$

In equilibrium, q_t and b_t^* are governed by the following two equations

$$q_t = \left[\phi_f \cdot \alpha_1(\sigma)\right] / \phi_d + \left\{\left[\phi_f \cdot \alpha_2(\sigma)\right] / \phi_d\right\} \cdot z_{t-1} , \qquad (13b)$$

$$b_t^* = \theta \cdot \alpha_1\left(\sigma\right) + \theta \cdot \alpha_2\left(\sigma\right) \cdot z_{t-1} . \qquad (13c)$$

Moreover, the endogenous growth rate of the supply of dollars in the domestic economy is given by

$$\left(Q_{t+1}/Q_t\right) = \left[\left(1+\sigma^*\right)\cdot z_{t+1}\right]/z_t \,, \tag{13d}$$

while the nominal exchange rate follows:

$$\left(e_{t+1}/e_t\right) = \left[\left(1+\sigma\right)\cdot z_t\right]/\left[\left(1+\sigma^*\right)\cdot z_{t+1}\right]. \tag{13e}$$

Finally, there are several conditions that characterize deposit contracts in equilibrium. One, the truth-telling condition in (2) holds. Two, the constraints on foreign credit must bind, and thus

$$d_{0,t} + d_{2,t+1} = f_0 \text{ and } d_{1,t+1} + d_{2,t+1} = f_1 . \tag{14a}$$

Three, the representative bank's long-term investment in equilibrium follows

$$k_t = \xi_1(\sigma) + \xi_2(\sigma)\cdot z_{t-1} . \tag{14b}$$

(14b) is a first order linear difference equation, and its reduced-form coefficients are given by $\xi_1(\sigma) \equiv f_0 + \left[\left(1-\phi_d-\phi_f\right)\cdot w\cdot \tilde{N}(\sigma)\right]/\mathfrak{M}(\sigma)$ and $\xi_2(\sigma) \equiv \left[\left(1-\phi_d-\phi_f\right)\cdot\theta\cdot\tilde{r}\cdot(1+\sigma)\right]/\mathfrak{M}(\sigma)$, where $\tilde{N}(\sigma) \equiv \{2+\theta\cdot\phi_d+\sigma\cdot[2-\phi_d\cdot(1-\theta)]\}$. Next, the withdrawals offered by banks to impatient and patient individuals in equilibrium are, respectively

$$c_{1,t} = \left[\left(f_1 - f_0\right)/\lambda\right] - \left[\left(r_0^* - 1\right)/\lambda\right]\cdot d_{0,t} , \tag{14c}$$

$$\begin{aligned} c_{2,t+1} = \; & Ж_0(\sigma) - \left(r_2^* - r_1^*\right)\cdot d_{2,t+1} + \frac{r_2^*\cdot\xi_2(\sigma)}{(1-\lambda)}\cdot z_{t-1} + \left[\frac{\phi_f}{(1-\lambda)\cdot\phi_d\cdot\left(1+\sigma^*\right)}\right]\cdot z_t \\ & + \frac{\phi_d\cdot w}{(1-\lambda)\cdot(1+\sigma)}\cdot\left(\frac{z_{t+1}}{z_t}\right) + \left[\frac{1}{(1-\lambda)\cdot(1+\sigma)}\right]\cdot z_{t+1} \end{aligned} \tag{14d}$$

where the intercept is given by $Ж_0(\sigma) \equiv \left[\frac{r_2^*\cdot\xi_1(\sigma)}{(1-\lambda)} + \frac{\phi_d\cdot w}{(1-\lambda)\cdot\left(1+\sigma^*\right)} - r_1^*\cdot f_1\right]$. Notice that (14d) is a second order, nonlinear difference equation in z_t, and contemporaneous in $d_{2,t+1}$. Last, we define the current account balance of this economy as[22]:

$$CA_t = \left(1-r_0^*\right)\cdot d_{0,t} + \left(1-r_1^*\right)\cdot d_{1,t} + \left(1-r_2^*\right)\cdot d_{2,t} + \left(d_{1,t}-d_{1,t+1}\right) + \left(d_{2,t}-d_{2,t+1}\right). \tag{14e}$$

Notice that the first three terms on the right-hand-side of (14e) are negative. The reduced-form equations for the triplet $\left(d_{0,t}, d_{1,t+1}, d_{2,t+1}\right)$ and, of course, the couple $\left(c_{1,t}, c_{2,t+1}\right)$ will depend, as we will see in the next section, on the particular set of equilibria it belongs to.

2.5 Multiplicity and Indeterminacy of Stationary Equilibria under Floating

In this section, we discuss the set of ***separating*** stationary equilibria. These equilibria are such that: 1) there are no misrepresentations of types; 2) there are no problems of liquidity, and 3) there is no early liquidation of the

[22] We follow strictly the guidelines of the Manual of Balance of Payments by the IMF.

long-term investment by banks[23]. Before we proceed we must point the following: there is what we call a *"core"* of variables that are independent of the foreign interest rates, and another set that contains foreign debt and state-contingent commodities that are determined as a result.

The Core in a Stationary Equilibrium. The core consists of a vector of five key variables: $\left(z_t, \tau_t, q_t, b_t^*, k_t\right)$. These five variables are always determinate in equilibrium, since they do not depend on the interest rates $\left(r_0^*, r_1^*, r_2^*\right)$. Interestingly, the core dynamic system is de-coupled, inheriting its dynamics from z_t. Notice that, under floating exchange rates, $\hat{x}$ represents the steady-state value of the variable x. Thus, the stationary core can be found in the five equations that follow:

$$\hat{z} = \alpha_1(\sigma)/[1-\alpha_2(\sigma)] = [\phi_d \cdot w \cdot (1+\sigma)/\ni(\sigma)], \tag{15a}$$

$$\hat{\tau} = (\hat{z}/\phi_d) - w = \langle w \cdot \{\theta \cdot \phi_d \cdot (\tilde{r}-1) + \sigma \cdot \phi_d \cdot [\theta \cdot (\tilde{r}-1)+1]\}/\ni(\sigma)\rangle, \tag{15b}$$

$$\hat{q} = \left(\phi_f \cdot \hat{z}/\phi_d\right) = \left[\phi_f \cdot w \cdot (1+\sigma)/\ni(\sigma)\right], \tag{15c}$$

$$\widehat{b^*} = \theta \cdot \hat{z} = [\theta \cdot \phi_d \cdot w \cdot (1+\sigma)/\ni(\sigma)], \tag{15d}$$

$$\hat{k} = \xi_1(\sigma) + \xi_2(\sigma) \cdot \hat{z}, \tag{15e}$$

where $\ni(\sigma) \equiv \sigma \cdot [1-\phi_d \cdot (1+\theta \cdot (\tilde{r}-1))] + 1 - \phi_d \cdot \theta \cdot (\tilde{r}-1)$. Notice that $\left(\hat{z}, \hat{q}, \widehat{b^*}\right)$ are increasing in the policy parameters (σ, ϕ_d, θ) and that, as expected, $\hat{q}$ is increasing in ϕ_f. In addition, $\hat{\tau}$ is nonlinear in both σ and ϕ_d but monotonically increasing in θ. Finally, $\hat{k}$ is increasing in σ, but nonlinear in $\left(\phi_d, \phi_f\right)$. With respect to the steady-state returns on domestic and foreign real money balances, the growth of the nominal exchange rate and the growth rate of the real exchange rate, they are all constant and equal to $(1+\sigma)^{-1}$, $\left(1+\sigma^*\right)$, $(1+\sigma)\cdot\left(1+\sigma^*\right)^{-1}$ and 1, respectively.

Foreign Debt in a Stationary Equilibrium. The amount that banks borrow from abroad is constant and non-negative in a stationary equilibrium, for all types of foreign debt-instrument, provided, of course that (10b) holds. Thus, the structure of foreign debt of a bank in a steady-state equilibrium is given by the triplet $\left(\widehat{d_0}, \widehat{d_1}, \widehat{d_2}\right) > 0$. This stationary debt structure vector permits us to calculate the current account balance in a stationary equilibrium:

$$\widehat{CA} = \left(1-r_0^*\right) \cdot \widehat{d_0} + \left(1-r_1^*\right) \cdot \widehat{d_1} + \left(1-r_2^*\right) \cdot \widehat{d_2} < 0 \tag{16}.$$

We must remark that the deficit of the current account in stationary equilibria poses significant doubt on the long-run viability of this economy, as one might expect.

A stationary equilibrium then is defined as the allocation $\left\{\left(\hat{z}, \hat{\tau}, q, \widehat{b^*}, \hat{k}\right), \left(\widehat{d_0}, \widehat{d_1}, \widehat{d_2}\right) \middle| \, \hat{l} = 0\right\} \in \mathbb{R}_{++}^5 \times \mathbb{R}_+^3$, which satisfies all the conditions given above. Of course, the allocation of state-contingent commodities $\left(\widehat{c_1}, \widehat{c_2}\right) \in \mathbb{R}_{++}^2$ follows directly by using (14c) and (14d). The

[23] In this section, we restrict our attention to equilibria where the conditions in the banks' problem guarantee no early liquidation in equilibrium.

particular type of equilibrium and its properties will depend on the composition of the vector $\left(\widehat{d_0}, \widehat{d_1}, \widehat{d_2}\right) > 0$. We discuss this issue as we go along.

Existence and Local Uniqueness of Stationary Equilibria. Before discussing fully the issue of existence, we must discuss the different types of equilibria that may arise[24], based on the properties of the structure of foreign debt issued by domestic banks. We will observe multiple stationary equilibria in this model economy with floating exchange rates. There are three cases, which we discuss below. The second subscript on variables denotes Case *j*, where $j = 1, 2, 3$.

Case 1: Equilibria with no intra-date debt d_0**.** The equilibria that belong to this case are stationary allocations characterized by the debt-structure $\left(\widehat{d_{0,1}}, \widehat{d_{1,1}}, \widehat{d_{2,1}}\right) = \left(0,\, f_1 - f_0,\, f_0\right)$. This allocation can be thought of as banks willing to borrow arbitrarily large values of $\widehat{d_{2,1}}$ but since foreign credit is rationed, banks must content themselves with $\widehat{d_{2,1}} = f_0$.

Case 2: Interior Solution of Debt-Structure. Any of these stationary allocations is characterized by a debt-structure of the form $\left(\widehat{d_{0,2}}, \widehat{d_{1,2}}, \widehat{d_{2,2}}\right) = \left(f_0 - \widehat{d_{2,2}},\, f_1 - \widehat{d_{2,2}}, \widehat{d_{2,2}}\right) \gg 0$. Notice that there is a continuum of allocations that satisfy this criterion. Of course, $0 < \widehat{d_{2,2}} < f_0$, $\widehat{d_{0,2}} = f_0 - \widehat{d_{2,2}} > 0$ and $\widehat{d_{1,2}} = f_1 - \widehat{d_{2,2}} > 0$ obtain.

Case 3: Equilibria with no long-term debt d_2**.** These equilibria are characterized by the debt-structure vector $\left(\widehat{d_{0,3}}, \widehat{d_{1,3}}, \widehat{d_{2,3}}\right) = \left(f_0, f_1 - f_0, 0\right)$. One could think of an explanation along the lines we used for Case 1: domestic banks are willing to borrow arbitrarily large amounts of intra-date debt, but they must content themselves with $\widehat{d_{0,3}} = f_0$.

The properties displayed by the stationary debt-structure in equilibrium depend, among other things, on the different values that the policy parameter σ --the growth of domestic money growth-- may take. The aforementioned properties have to do with existence, the number of equilibria, and the case to which the equilibrium belongs to. Thus, σ is a bifurcation parameter of the steady-state allocation $\left\{\left(\hat{z}, \hat{\pi}, q, \hat{b}^*, \hat{k}\right), \left(\widehat{d_{0,j}}, \widehat{d_{1,j}}, \widehat{d_{2,j}}\right) \middle|\; l = 0\right\}$, and so is the structure of the interest rates $\left(r_0^*, r_1^*, r_2^*\right) \gg 1$.

Proposition 1. Define the set $\mathcal{P} = \left\{\sigma, r_0^*, r_1^*, r_2^*\right\} \in \mathbb{R}^4$ to be the space of bifurcation parameters under a floating exchange rate regime. Bifurcation values of these parameters partition $\mathcal{P}$ into three subsets with defining characteristics that we describe below.

Subset 1= $\mathcal{P}_1$*:* Existence of Case 1 Equilibria. Given $\hat{\varepsilon}$ as defined in the Appendix, the two mutually exclusive conditions must hold for equilibria of Case 1 to exist:

- <u>Condition 1</u>: $\hat{\varepsilon} > 0$ must hold.
- <u>Condition 2:</u> when $\hat{\varepsilon} < 0$ obtains, $\sigma < \hat{\sigma} \equiv \left[\hat{\delta} + \lambda \cdot \phi_d \cdot w / (-\varepsilon)\right]$ must hold

Subset 2= $\mathcal{P}_2$*:* Existence of Case 2 Equilibria. This type of equilibrium always exists. Therefore, $\mathcal{P}_2 = \mathcal{P}$.

Subset 3= $\mathcal{P}_3$*:* Existence of Case 3 Equilibria. Given the expressions *A*, *B* and *C*, as defined in the Appendix, equilibria of Type 3 exist when

- <u>Condition 3:</u> $\max\left\{A, B\right\} < r_1^* \le C$

[24] This general classification will also apply to dynamic equilibria, as we will see in the next section.

Proof: This proof is very complex and lengthy. It is available upon request.

Table 2 summarizes the results on existence of stationary equilibria, and presents as well how the scope of existence varies with σ . It is apparent that the stationary allocations of state-contingent commodities $\left(\widehat{c_{1,j}}, \widehat{c_{2,j}}\right)$ follow directly from $\left(\widehat{d_{0,j}}, \widehat{d_{1,j}}, \widehat{d_{2,j}}\right)$.

The issue of multiple equilibria raises a new set of questions related to the properties of local uniqueness and determinacy, which focus on the mapping from allocations to prices in equilibrium. The practice in standard General Equilibrium theory is the construction of a particular economic environment with the aim of ensuring that the equilibria are *"regular,"* which means: 1) the number of equilibria is finite, and, 2) there is a one-to-one mapping between the vectors of relative prices and the excess demand function in a neighborhood of the equilibrium allocation. However, our model economy violates the two conditions of regularity. First, there is typically a continuum of equilibria. Second, the mapping between the vectors of relative prices and the excess demand *correspondence* is not one-to-one. Thus, steady-state equilibria in our economy are *"irregular,"* and thus, they are not locally unique nor they are determinate.

Let us illustrate the nature of the irregularity in our model economy. Notice that the core in the steady-state $\left(\hat{z}, \hat{\tau}, q, \widehat{b^*}, \hat{k}\right)$ is always unique and determinate, since it is not associated with the vector of foreign interest rates. However, given a fixed point in the parameter-space, for each stationary debt-structure vector $\left(\widehat{d_{0,j}}, \widehat{d_{1,j}}, \widehat{d_{2,j}}\right)$ there is typically a continuum of vectors of interest rates $\left(r_0^*, r_1^*, r_2^* = R\right)$ that satisfy the equilibrium conditions. To fix ideas, let us discuss briefly the equilibrium in Case 1, in which this issue appears to be a little simpler. In this case, the debt-structure $(0, f_1 - f_0, f_0)$ is unique. However, as the reader can observe in Figure 1, there is a continuum of vector prices $\left(r_0^*, r_1^*, r_2^* = R\right)$ that satisfy the equilibrium conditions, and, thus, are associated with the allocation $(0, f_1 - f_0, f_0)$. This is illustrated by the thick gray line on the plane $r_2^* = R$, which defines the set of possible equilibrium price vectors. The range of $\left(r_0^*, r_1^*, r_2^*\right)$ in an equilibrium that belongs to Case 1 depends on the following three boundary functions[25] : $\tilde{A}(\sigma) \equiv r \cdot \tilde{B}(\sigma) \cdot \left[\lambda \cdot r + (1-\lambda) \cdot r_0^*\right]^{-1}$, $\tilde{B}(\sigma) \equiv \bar{a} \cdot \left(f_1 - f_0\right)^{-1}$ and $\tilde{C}(\sigma) \equiv \left[\tilde{B}(\sigma) - (1-\lambda) \cdot r\right] \cdot \lambda^{-1}$. Stationary equilibria exist when $\tilde{A}(\sigma) < r_1^* < \tilde{B}(\sigma) < \tilde{C}(\sigma)$ holds[26].

Turning back to the general case, we must emphasize that the indeterminacy of equilibria goes two ways: 1) for a given vector $\left(\widehat{d_{0,j}}, \widehat{d_{1,j}}, \widehat{d_{2,j}}\right)$ there is a continuum of vectors $\left(r_0^*, r_1^*, r_2^*\right)$ consistent with equilibrium conditions; and 2) for a given vector $\left(r_0^*, r_1^*, r_2^*\right)$, there may be more than one vector $\left(\widehat{d_{0,j}}, \widehat{d_{1,j}}, \widehat{d_{2,j}}\right)$ consistent with equilibrium. Moreover, these vectors may belong to each of the different cases. As an example, it is possible that one vector may belong to Case 1, one or more to Case 2 and another one to Case 3. Proposition 1 below illustrates the general properties of separating steady-state equilibria in our model economy.

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[25] These conditions are simplifications of Conditions 1 and 2.
[26] This is a simplification of conditions 1 and 2.

2.6 Dynamic Equilibria under a Floating Exchange Rate Regime

The dynamic system for this economy has three parts: the core dynamic system, the dynamic system for the debt-structure vector $\left(d_{0,t}, d_{1,t+1}, d_{2,t+1}\right)$, and the dynamic system for the space-contingent commodities $\left(c_{1,t}, c_{2,t+1}\right)$. Obviously, the latter will depend on the arrangement of foreign debt together with the particular case the equilibrium belongs to. We illustrate the configuration of causality relationships of the dynamic system in Figure 2.

2.6.1 The Core Dynamic System with Floating. The core dynamic system in equilibrium involves the five core variables $\left(\widehat{z}_t, \widehat{\tau}_t, \widehat{q}_t, \widehat{b}_t^*, \widehat{k}_t\right)$ and it consists of equations (12c), (12d), (13b), (13c) and (14b). We must mention that this is a de-coupled system where all dynamics originates from the real balances of *wons*, $\widehat{z}_t$, in equation (12c). Proposition 2 discusses the main dynamic properties of the core system.

Proposition 2 Under floating exchange rates, the quintet $\left(\hat{z}_t, \hat{\tau}_t, \hat{q}_t, \hat{b}_t^*, \hat{k}_t\right)$ displays monotonic dynamics along the equilibrium paths around the stationary core. Let $i = 1,2,3,4,5$ index the variables in the core and their associated eigenvalues. All eigenvalues $\hat{\epsilon}_i\left(\sigma\right)$ are smooth functions of all the policy parameters, and four non-overlapping cases may arise: there is no dynamics, the monotonic dynamics is stable, the dynamics display unit roots or there is monotonic divergence. In particular, we find:

i) For a fixed combination of returns, the policy parameters $\left(\phi_d, \phi_f, \theta\right)$ and $\forall i$, $\hat{\epsilon}_i\left(\sigma\right)$ is a monotonically increasing and concave function of the rate of domestic money growth, σ. Moreover, all eigenvalues have a bifurcation value at $\tilde{\sigma} \equiv \left(2-\tilde{r}\right)/\left(\tilde{r}-1\right) > 0$ such that $\hat{\epsilon}_i\left(\tilde{\sigma}\right) = 1$. Moreover, for $\sigma > \left(<\right)\tilde{\sigma}$, dynamic equilibria are stable (unstable).

ii) The interaction between all the policy parameters and the nature of the dynamics is highly complex. Interestingly, there are many different sub-regions in the parameter-space that determine characteristic dynamic properties. As an example, we illustrate the case of the eigenvalue $\hat{\epsilon}_1\left(\sigma\right) = \alpha_2\left(\sigma\right)$ in Table 3[27]. The reader may see that there are many different restrictions/combinations of parameters that yield monotonic dynamics, a unit root or monotonic divergence.

2.6.2 The Dynamic System of the Debt-Structure under Floating. We now turn to discuss the debt-structure vector in dynamic equilibria that result from a floating exchange rate regime. Below, we start our discussion of each case.

Foreign-Debt Dynamics in Cases 1 and 3: the equilibrium debt-structure in either of these extreme cases is stationary. On the first hand, the composition of foreign credit in Case 1 is given by $\left(\widehat{d_{0,t,1}}, \widehat{d_{1,t+1,1}}, \widehat{d_{2,t+1,1}}\right) = \left(\widehat{d_{0,1}}, \widehat{d_{1,1}}, \widehat{d_{2,1}}\right) = \left(0, f_1 - f_0, f_0\right), \forall t \geq 1$, and the corresponding current account $\widehat{CA}_1 = \left(1 - r_1^*\right) \cdot f_1 + \left(r_1^* - r_2^*\right) \cdot f_0 < 0$ displays a nontrivial deficit. On the other hand, in Case 3, foreign credit is given by $\left(\widehat{d_{0,t,3}}, \widehat{d_{1,t+1,3}}, \widehat{d_{2,t+1,3}}\right) = \left(\widehat{d_{0,3}}, \widehat{d_{1,3}}, \widehat{d_{2,3}}\right) = \left(f_0, f_1 - f_0, 0\right), \forall t \geq 1$, while there is also a deficit in the associated current account balance $\widehat{CA}_3 = \left(1 - r_1^*\right) \cdot f_1 + \left(r_0^* - r_1^*\right) \cdot f_0 < 0$.

[27] Due to limited space, we only include the results for this eigenvalues. The complete results are available upon request.

Foreign-Debt Dynamics in Case 2: the debt-structure vector displays non trivial dynamics. We present the results for the long-term, foreign-debt instrument[28]. The dynamics of $\widehat{d_{2,t+1,2}}$ is governed by

$$\widehat{d_{2,t+1,2}} = \Theta_0(\sigma) + \Theta_1(\sigma) \cdot z_{t-1} + \Theta_2(\sigma) \cdot (z_{t+1}/z_t) + \Theta_3(\sigma) \cdot z_{t+1} + \Theta_4(\sigma) \cdot z_t. \quad (17)$$

Equation (17) is a second order, nonlinear, difference equation in z_t. The reduced-form coefficients are given by $\Theta_0(\sigma) \equiv r_2^* \cdot \xi_1(\sigma) + [\phi_d \cdot w/(1+\sigma^*)] - r_1^* \cdot (f_1 - r_0^* \cdot f_0)$, $\Theta_1(\sigma) \equiv r_2^* \cdot \xi_2(\sigma)$, $\Theta_2(\sigma) \equiv [\phi_d \cdot w/(1+\sigma^*)]$, $\Theta_3(\sigma) \equiv (1+\sigma)^{-1}$ and $\Theta_4(\sigma) \equiv \phi_f \cdot [\phi_d \cdot (1+\sigma^*)]^{-1}$. We proceed by first augmenting the state-space by using $y_{t+1} = z_t$, which reduces the order of the system: it becomes a first order dynamic system in (z_t, y_t).

<u>Local Stability Analysis.</u> We took a First Order Taylor approximation around the steady-state. The trace and determinant of the Jacobian matrix associated with this system in the steady-state are continuous monotonic functions of the parameters, and given by the expressions:

$$Tr(J) = \frac{\phi_d \cdot w}{(\phi_d \cdot w + \hat{z})} - \frac{(1+\sigma) \cdot \phi_f \cdot \hat{z}}{\phi_d \cdot (1+\sigma^*) \cdot (\phi_d \cdot w + z)} \quad (18a)$$

$$Det(J) = \frac{(1+\sigma) \cdot r_2^* \cdot \xi_2(\sigma) \cdot \hat{z}}{(\phi_d \cdot w + \hat{z})} > 0. \quad (18b)$$

It is apparent, from (18b), that the pair of eigenvalues has the same sign. Notice that $Tr(J)$ is a monotonically decreasing function of σ, $Det(J)$ is a monotonically increasing function of σ, while the discriminant $\Delta \equiv [Tr(J)]^2 - 4 \cdot Det(J)$ and eigenvalues are highly nonlinear in the same parameter. Moreover, the world inflation rate and the policy parameters interact with the rate of domestic money growth, altering the dynamic properties of the system. We now describe how the dynamic properties of the long-term debt and bifurcations change as we contemplate scenarios with different combinations of parameters[29]. In particular, we present first the "*baseline sequence*" in Proposition 3, which describes how the dynamic properties of the long-term debt vary with the rate of domestic money growth in the *baseline scenario*. Next, for $(\sigma^*, \phi_d = \phi_f, \theta)$ and $(\tilde{r}, r_0^*, r_1^*, r_2^*) \gg 1$, we describe how the *baseline sequence* changes with different values of a particular parameter. For our convenience, we define the following notation: (+)sink is a sink with positive eigenvalues, (-)sink is a sink with negative eigenvalues, (-)saddle is a saddle with negative eigenvalues, (-)source is a source with negative eigenvalues, (+)complex-stable indicates complex conjugates with a positive real part that is less than one, (-)complex-stable indicates complex conjugates with a negative real part that is less than one, and (-)complex-unstable indicates complex conjugates with a real part that is outside the unit circle.

[28] The results for $d_{0,t,2} = f_0 - d_{2,t+1,2}$ and $d_{1,t+1,2} = f_1 - d_{2,t+1,2}$ follow directly.

[29] Due to limited space, we only present a summary of the results of our simulations. The detailed results and the simulation files are available upon request. The parameter values used in the *baseline scenario* are: $w = 2$, $\phi_d = \phi_f = 0.1$, $\theta = 0.2$, $\lambda = 0.2$, $\sigma^* = 0.05$, $\tilde{r} = 1.1$, $R = r_2^* = 1.2$, $r_0^* = 1.08$ and $r_1^* = 1.11$. The *baseline scenario* only represents a reasonable starting point, since this is <u>not</u> a calibration exercise.

Proposition 3 The *baseline sequence* as a function of σ under floating exchange rates consists of: (+)sink, (+)complex-stable, (-)complex-stable, (-)complex-unstable, (-)source and (-)saddle. This means that for values of σ that are low enough, the steady-state $\hat{d}_2$ is a sink with positive eigenvalues; as σ increases, the eigenvalues become complex conjugates with a positive and stable real part that next turns into a negative stable real part, displaying cyclical and non-cyclical stable fluctuations; with higher values of σ, the real part of complex conjugates become negative and unstable, displaying unstable cyclical oscillations; as σ continues to increase, the steady-state becomes a source with large and negative eigenvalues, dynamic equilibria are unstable and display large and exploding non-cyclical fluctuations along dynamic paths; finally, for values of the rate of domestic money growth that are high enough, the steady state becomes a saddle with non-cyclical stable fluctuations along the stable manifold. The scope for determinacy dominates, but so does the scope for endogenously arising volatility.

Proposition 3 illustrates the rich dynamics of long-term debt in Case 2. Market-generated volatility is always present and dominates the dynamic paths. We now describe how the sequence just described varies with different parameters. Because of their length, we do not present the following results in a Proposition.

2.6.3 The Dynamic System of the Space-Contingent Commodities. We now turn to discuss the properties of the pair $\left(\widehat{c_{1,t,j}}, \widehat{c_{2,t+1,j}}\right) \gg 0$ in dynamic equilibria with floating exchange rates. This dynamic system consists of equations (14c) and (14d), which inherit their dynamics from the dynamic system of the debt-structure. We first discuss the dynamic properties of consumption by impatient agents for the different cases of equilibria, and we follow with the analysis of the consumption by patient agents. Below, we start our discussion of each case.

Dynamics of the Consumption by Impatient Agents. In general, the consumption by impatient agents is given by $\lambda \cdot \widehat{c_{1,t,j}} = f_1 - f_0 - \left(r_0^* - 1\right) \cdot \widehat{d_{0,t,j}}$. In particular, for each case, we obtain

$$\lambda \cdot \widehat{c_{1,t,1}} = f_1 - f_0, \text{ for } j = 1, \tag{19a}$$

$$\lambda \cdot \widehat{c_{1,t,2}} = f_1 - r_0^* \cdot f_0 + \left(r_0^* - 1\right) \cdot \widehat{d_{2,t+1,2}}, \text{ for } j = 2, \tag{19b}$$

$$\lambda \cdot \widehat{c_{1,t,3}} = f_1 - r_0^* \cdot f_0, \text{ for } j = 3. \tag{19c}$$

Notice that $\widehat{c_{1,t,1}} = \widehat{c_{1,1}}$ and $\widehat{c_{1,t,3}} = \widehat{c_{1,3}}$, $\forall t \geq 1$ and thus, the consumption by patient individuals is always stationary in Cases 1 and 3. However, in Case 2 the evolution of $\widehat{c_{1,t,2}}$ is governed by the term $\left(r_0^* - 1\right) \cdot \widehat{d_{2,t+1,2}}$, which indicates non-trivial dynamics, according to our discussion in the previous section. Specifically, the dynamic properties of $\widehat{c_{1,t,2}}$ are very similar to the dynamics of $\widehat{d_{2,t+1,2}}$.

Dynamics of the Consumption by Patient Agents. The dynamic behavior of $\widehat{c_{2,t+1,j}}, \forall i$, is identical to that of long-term debt in Case 2, regardless of the Case the equilibrium belongs to. This is apparent from comparing equation (17) against the following equation:

$$\widehat{c_{2,t+1,j}} = \Theta_{0,j}\left(\sigma\right) + \Theta_1\left(\sigma\right) \cdot \widehat{z_{t-1}} + \Theta_2\left(\sigma\right) \cdot \left(\widehat{z_{t+1}} / \widehat{z_t}\right) + \Theta_3\left(\sigma\right) \cdot \widehat{z_{t+1}} + \Theta_4\left(\sigma\right) \cdot \widehat{z_t}, \tag{19d}$$

where only the intercept varies across the different cases such that $\Theta_{0,1}\left(\sigma\right) \equiv r_2^* \cdot \xi_1\left(\sigma\right) + \phi_d \cdot w \cdot \left(1 + \sigma^*\right)^{-1} + r_1^* \cdot \left(f_1 - f_0\right) - r_2^* \cdot f_0$,

$\Theta_{0,2} \equiv r_2^* \cdot \xi_1(\sigma) + \phi_f \cdot w \cdot (1+\sigma^*)^{-1} - r_1^* \cdot f_1 - (r_2^* - r_1^*) \cdot \Theta_0(\sigma)$, and $\Theta_{0,3}(\sigma) \equiv r_2^* \cdot \xi_1(\sigma) + \phi_f \cdot w \cdot (1+\sigma^*)^{-1} - r_1^* \cdot f_1$. Obviously, (18a) and (18b) still apply. Proposition 4 summarizes the results for the pair of state-contingent commodities $\left(\widehat{c_{1,t,j}}, \widehat{c_{2,t+1,j}}\right)$.

Proposition 4 The stationary pair $\widehat{c_{1,1}}$ and $\widehat{c_{1,3}}$ takes up from the also stationary values $\widehat{d_{2,1}} = f_0$ and $\widehat{d_{2,1}} = 0$, respectively, while $\widehat{c_{1,2}}$ inherits its dynamics from $\widehat{d_{2,t+1,2}}$. Regarding the triplet of case-contingent consumption by patient agents $\left(\widehat{c_{2,t+1,1}}, \widehat{c_{2,t+1,2}}, \widehat{c_{2,t+1,3}}\right)$, they come into their dynamics from $\widehat{d_{2,t+1,2}}$. In particular, each of the variables $\widehat{c_{2,t+1,1}}$, $\widehat{c_{2,t+1,2}}$ and $\widehat{c_{2,t+1,3}}$ share the same dynamic properties than $\widehat{d_{2,t+1,2}}$, and they also share the same baseline sequence.

Summary: We now summarize the results obtained along dynamic paths. There is a nontrivial scope for complex eigenvalues that contributes to both cyclical and non-cyclical fluctuations; in some cases, the fluctuations can be significantly large and explosive. The scope for stability --and indeterminacy-- is typically small, and the scope for determinacy typically dominates, but fluctuations are observed on the stable manifold. Moreover, unstable and oscillating divergence is observed in general. The reserve requirements play the role of stabilizing, at least partially, the dynamic equilibria in this model economy, while backing the domestic supply plays the opposite role. Thus, these results provide us with policy recommendations: to implement high and binding reserve requirements, but keeping the backing of the money supply to a minimum. In the extreme case of $\theta = 0$, the order of the dynamic system is reduced, which one can interpret as the ultimate stabilization of dynamic equilibria.

3. Fixed Exchange Rates: the Case of Malaysia

In this section, we consider a small, open economy that replicates the case of Malaysia at the time of the crisis[30]. This economy is identical to the economy discussed in Section 2 in every respect but the exchange rate regime and associated monetary policy: this second model economy operates under a fixed rather than a flexible exchange rate regime. We still focus on separating equilibria with truth-telling where crises cannot occur. As in Hernandez-Verme (2004), we construct this fixed exchange rate regime such that a currency board emerges as a special case[31], and we restrict our attention to the study of equilibria under a very hard peg where the fixed exchange rate remains constant, as opposed to intermediate pegs. We continue to use the *won* as the domestic currency and the *US dollar* as the foreign, international currency.

3.1 Monetary Policy under Fixed Exchange Rates

[30] It is possible to argue as well that the model in this section not only represents the Malaysian economy, but also the Hong-Kong economy. We do not take a stand in this respect.

[31] A currency board requires two elements: a fixed exchange rate regime and $\theta = 1$.

In this model economy with a very hard peg, the monetary authority has different priorities as to the aspects of monetary policy rules to be implemented. In the first place, policymakers worry about the setting, once-and-for-all, of the nominal exchange rate at the initial date. Second, the maintenance and sustainability of the peg by choosing the appropriate backing of the domestic money supply with holdings of foreign-reserve assets. Notice that the policy of binding reserve requirements is still in place.

With respect to the setting of the nominal exchange rate, we let $e > 0$ denote, without loss of generality, the time-invariant and exogenous nominal exchange rate set by the monetary authority at the initial date, t=0[32]. It is essential, under this policy, that the central bank must stand ready to buy or sell foreign currency at the fixed exchange rate, causing the domestic money supply to change as needed to meet this target. The latter implies that the nominal supply of domestic currency is endogenous: by fixing the nominal exchange rate, the monetary authority gives up full control of M_t, and we say that the monetary policy is subordinated[33]. The withdrawals/injection of domestic currency (*wons*), as in Section 2, are accomplished through the monetary lump-sum transfers in the amount of τ_t goods to the *ex ante* identical young agents There are many reasons that explain the choice of a fixed nominal exchange rates regime in spite of the lack of an independent monetary policy. Among them, we must mention that, under a very hard peg, the domestic country inherits the world inflation rate, contributing to a relatively quick stabilization and reduction of domestic rates of inflation. This also helps promote the credibility of this policy, in general, and the monetary authority, in particular, among the general public[34].

The monetary authority may also choose to hold reserves in the form of interest-bearing, foreign-reserve assets. This rule takes the same form than in Section 2. However, the general purpose of this policy rule is somehow different under fixed exchange rates. These reserve-holdings aim to back the dollar-value of the domestic money supply but with the purpose instead of meeting the needs of the public who wants to buy or sell foreign currency at the fixed exchange rate e, so that speculative attacks on the domestic currency can be avoided or at least minimized. Thus, this rule also contributes to the sustainability of the very hard peg in place. We continue to use the same notation in as much as possible. Notice that, at the same time at date 0, the monetary authority sets both e and θ. Next, we modify equation (8) accordingly, and obtain

$$B_t^* = \theta \cdot \left(\frac{M_t}{e} \right). \tag{20}$$

[32] We must point out that our goal in this paper is *not* the choice of the "optimal" nominal exchange rate.
[33] The reader may recall "Tinbergen's principle": for each policy goal, one policy tool is needed. This principle applies very well to this case. In general, one cannot hope to attend to two or more different goals by using only one policy tool to achieve both. In this particular case, the monetary authority shall not expect to keep the nominal exchange rate fixed and constant, and, at the same time, keep the control of the money supply, since this would imply that there are two goals but only one monetary policy tool available (the money supply.)
[34] The reader may recall the on-going debate on exchange rate regimes: rules versus discretion and credibility versus flexibility.

As we mentioned before, a currency board arrangement obtains as a particular case, when the monetary authority sets $\theta = 1$ at $t = 0$. In what follows, we study the general case where $\theta \in [0,1]$.

The new regime of monetary and foreign exchange policy thus defined imposes different restrictions on the resources effectively available to the monetary authority. For instance, the nominal supply of domestic currency is endogenous in that it adjusts to keep the nominal exchange rate at the level e set by the government. In addition, a change in the foreign-reserve position of the monetary authority affects the resources available to the government as well as in the case of floating exchange rates. Of course, the monetary authority must account for these two sources of resources, resulting in the following budget constraint:

$$\tau_t = \frac{M_t - M_{t-1}}{p_t} - \frac{B_t^* - \tilde{r} \cdot \left(p_t^* / p_{t-1}^* \right) \cdot B_{t-1}^*}{p_t^*} = \phi_d \cdot \left(w + \tau_t \right) - \left(p_{t-1} / p_t \right) \cdot \phi_d \left(w + \tau_{t-1} \right) - \left(b_t^* - \tilde{r} \cdot b_{t-1}^* \right) . \quad (21)$$

The first term on the right hand side of equation (21) represents the real value of any changes in the nominal supply needed to sustain the fixed nominal exchange rate. The second term indicates the effects of any changes in the real foreign-reserve position of the government.

3.2 General Equilibrium in a Fixed Exchange Rate Regime

It is also true that the general equilibrium under fixed exchange rates is very complex. We try to simplify our presentation, and, thus, we continue to proceed by groups, as we did in Section 2, while we substitute equations when necessary.

First, the restrictions on international transactions, equations (10a) and (10b) still hold: purchasing power parity and no arbitrage, respectively. Second, domestic and foreign currencies must be dominated in rate of return for the reserve requirements to be binding, implying that (11a) and (11b) still hold.

Third, regarding the market for *wons*, the market clearing condition (12a) still holds. However, the rate of return on real won balances changes significantly under a hard peg, and we substitute equation (12b) by the equation

$$\left(p_t / p_{t+1} \right) = \left(p_t^* / p_{t+1}^* \right) = \left(1 + \sigma^* \right)^{-1} . \quad (22b)$$

This last equation implies that the domestic country inherits the world inflation rate, as we mentioned in the previous section. However, notice that σ^* is not a parameter under the control of the monetary authority, reflecting the lack of control of the domestic money supply. Accordingly, the equilibrium laws of motion in equations (12c) and (12d) must be modified as well, and the following two equations obtain:

$$\tau_t = \eta_1\left(\sigma^* \right) + \eta_2\left(\sigma^* \right) \cdot \tau_{t-1} \quad (22c)$$

$$z_t = \rho_1\left(\sigma^* \right) + \rho_2\left(\sigma^* \right) \cdot \tau_{t-1} . \quad (22d)$$

The reduced-form coefficients above are defined as: $\eta_1\left(\sigma^* \right) \equiv \left\langle \phi_d \cdot w \cdot \left\{ \left(1 + \sigma^* \right) \cdot \left[\theta \cdot \left(\tilde{r} - 1 \right) + 1 \right] - 1 \right\} / \text{И}\left(\sigma^* \right) \right\rangle$, $\eta_2\left(\sigma^* \right) \equiv \left\{ \phi_d \cdot \left[\theta \cdot \tilde{r} \cdot \left(1 + \sigma^* \right) - 1 \right] / \text{И}\left(\sigma^* \right) \right\}$, $\rho_2\left(\sigma^* \right) \equiv \phi_d \cdot \eta_2\left(\sigma^* \right)$ and $\rho_1\left(\sigma^* \right) \equiv \phi_d \cdot \left[w + \eta_1\left(\sigma^* \right) \right]$, where

$И(\sigma^*) \equiv (1+\sigma^*) \cdot [1-\phi_d \cdot (1-\theta)]$. Notice that equations (22c) and (22d) are first order linear difference equations in τ_t. We must remark again that, under this hard peg, the dynamics of the system originates in τ_t instead of z_t, as it was the case under floating exchange rates.

We now turn to the market for foreign currency in our fourth group. Obviously, the market clearing condition (13a) still holds, but we must modify the equilibrium laws of motion in equations (13b) and (13c) to represent the hard peg instead. Thus, the following two equilibrium laws of motion obtain:

$$q_t = \chi_1(\sigma^*) + \chi_2(\sigma^*) \cdot \tau_{t-1} \tag{23b}$$

$$b_t^* = \psi_1(\sigma^*) + \psi_2(\sigma^*) \cdot \tau_{t-1}. \tag{23c}$$

The coefficients in (23b) and (23c) are defined as $\chi_1(\sigma^*) \equiv \phi_f \cdot \rho_1(\sigma^*) / \phi_d$, $\chi_2(\sigma^*) \equiv \phi_f \cdot \rho_2(\sigma^*) / \phi_d$, $\psi_1(\sigma^*) \equiv \theta \cdot \rho_1(\sigma^*)$ and $\psi_2(\sigma^*) \equiv \theta \cdot \rho_2(\sigma^*)$. Next, the rate of growth of the supply of foreign currency in circulation in the domestic economy (Q_t) remains unchanged and governed by (13d), while now equation (13e) becomes

$$(e_{t+1}/e_t) = (e/e) = 1. \tag{23d}$$

The previous equation reflects the fixed and time invariant nominal exchange rate in this policy regime.

The last group has to do with the deposit contract offered by banks under this regime. One, the self-selection constraint in (2) holds. Two, the constraint on foreign credit still binds and thus (14a) still holds. Three, the equilibrium law of motion for the long-term investment in this new regime is now given by

$$k_t = \varsigma_1(\sigma^*) + \varsigma_2(\sigma^*) \cdot \tau_{t-1}, \tag{24a}$$

where $\varsigma_1(\sigma^*) \equiv f_0 + (1-\phi_d-\phi_f) \cdot \rho_1(\sigma^*) / \phi_d$ and $\varsigma_2(\sigma^*) \equiv (1-\phi_d-\phi_f) \cdot \rho_2(\sigma^*) / \phi_d$. Four, the total return on domestic currency reserves and foreign currency reserves under this policy regime are given, respectively, by the following two equations:

$$\phi_d \cdot (p_t / p_{t+1}) \cdot (w+\tau_t) = \mu_1(\sigma^*) + \mu_2(\sigma^*) \cdot \tau_{t-1} \tag{24b}$$

$$\phi_f \cdot (p_t^* / p_{t+1}^*) \cdot (w+\tau_t) = \upsilon_1(\sigma^*) + \upsilon_2(\sigma^*) \cdot \tau_{t-1}, \tag{24c}$$

where the coefficients are $\mu_1(\sigma^*) \equiv \rho_1(\sigma^*) / (1+\sigma^*)$, $\mu_2(\sigma^*) \equiv \rho_2(\sigma^*) / (1+\sigma^*)$, $\upsilon_1(\sigma^*) \equiv \chi_1(\sigma^*) / (1+\sigma^*)$ and $\upsilon_2(\sigma^*) \equiv \chi_2(\sigma^*) / (1+\sigma^*)$. Five, the space-contingent commodities are governed by (14c) and

$$(1-\lambda) \cdot c_{2,t+1,j} + r_1^* \cdot d_{1,t+1,j} + r_2^* \cdot d_{2,t+1,j} = \omega_1(\sigma^*) + \omega_2(\sigma^*) \cdot \tau_{t-1}. \tag{24b}$$

In the latter equation, the parameters are $\omega_1(\sigma^*) \equiv r_2^* \cdot \varsigma_1(\sigma^*) + \mu_1(\sigma^*) + \upsilon_1(\sigma^*)$ and $\omega_2(\sigma^*) \equiv r_2^* \cdot \varsigma_2(\sigma^*) + \mu_2(\sigma^*) + \upsilon_2(\sigma^*)$. Six and last, the definition of the current account balance given in equation (14e) still holds.

We want to highlight two issues, at this time: 1) all the reduced-form coefficients are a function now of σ^*, but σ^* is determined in the world markets; 2) the structure of the general equilibrium system under fixed exchange rates shares the general structure of causality. In particular, all the dynamics originates from

$\overline{\tau}_{t-1}$, and we observe the same set of core variables[35] $\left(\overline{\tau}_t, \overline{z}_t, \overline{q}_t, \overline{b}_t^*, \overline{k}_t\right)$ that will affect the composition of standing foreign-debt $\left(\overline{d}_{0,t,j}, \overline{d}_{1,t+1,j}, \overline{d}_{2,t+1,j}\right)$ and, thus, also the space-contingent commodities $\left(\overline{c}_{1,t,j}, \overline{c}_{2,t+1,j}\right)$. Figure 3 below illustrates these details, in order to facilitate the reader's comprehension and insight of this second model economy.

3.3 Multiplicity and Indeterminacy of Stationary Equilibria under a Hard Peg

In this section, we discuss the set of ***separating*** stationary equilibria that obtain from a fixed exchange regime. In these equilibria, all agents behave according to their true type and there are no problems of liquidity. We will proceed by first characterizing the core in steady-state equilibria. Next, we characterize the stationary debt-structure vectors and the steady-state vectors, which define the stationary space-contingent commodities.

3.3.1 The Core in the Steady-State Equilibrium. The five variables that belong to the core, $\left(\overline{\tau}_t, \overline{z}_t, \overline{q}_t, \overline{b}_t^*, \overline{k}_t\right)$, are also determinate under a fixed exchange rate regime whenever an equilibrium exists, since they do no depend on the foreign interest rates $\left(r_0^*, r_1^*, r_2^*\right)$. We impose stationarity on equations (22c), (22d), (23b), (23c) and (24a), and obtain the steady-state values for the variables in the core that we display in the following five expressions:

$$\overline{\tau} = \eta_1\left(\sigma^*\right) / \left[1-\eta_2\left(\sigma^*\right)\right] = \left\langle \phi_d \cdot w \cdot \left\{ \left(1+\sigma^*\right) \cdot \left[1+\theta \cdot \left(\tilde{r}-1\right)\right] - 1 \right\} / \mathrm{M}\left(\sigma^*\right) \right\rangle, \quad (25a)$$

$$\overline{z} = \left[\phi_d \cdot w \cdot \left(1+\sigma^*\right) / \mathrm{M}\left(\sigma^*\right)\right], \quad (25b)$$

$$\overline{q} = \left[\phi_f \cdot w \cdot \left(1+\sigma^*\right) / \mathrm{M}\left(\sigma^*\right)\right], \quad (25c)$$

$$\overline{b^*} = \left[\theta \cdot \phi_d \cdot w \cdot \left(1+\sigma^*\right) / \mathrm{M}\left(\sigma^*\right)\right], \quad (25d)$$

$$\overline{k} = f_0 + \left(1-\phi_d-\phi_f\right) \cdot w + \left\langle \phi_d \cdot \left(1-\phi_d-\phi_f\right) \cdot \left\{ \left(1+\sigma^*\right) \cdot \left[\theta \cdot \left(\tilde{r}-1\right)+1\right] - 1 \right\} / \mathrm{M}\left(\sigma^*\right) \right\rangle. \quad (25e)$$

Here, $\mathrm{M}\left(\sigma^*\right) \equiv \left(1+\sigma^*\right) - \phi_d \cdot \left\{\left(1+\sigma^*\right) \cdot \left[1+\theta \cdot \left(\tilde{r}-1\right)\right] - 1\right\}$ and $\overline{x}$ denotes the stationary value of the variable x under a fixed exchange rate regime. It transpires that $\left(1+\sigma^*\right) \cdot \left[1+\theta \cdot \left(\tilde{r}-1\right)\right] > 1$, $\forall \sigma^* > -1$. Given the latter, we find it reasonable to restrict our attention to allocations where $\left(1+\sigma^*\right) > \phi_d \cdot \left\{\left(1+\sigma^*\right) \cdot \left[1+\theta \cdot \left(\tilde{r}-1\right)\right] - 1\right\} > 0$ holds $\forall \sigma^* > -1$. It follows, from (25a,) that $\overline{\tau} > 0$, and $\left(\partial \overline{\tau} / \partial \sigma^*\right) > 0$ obtains.

3.3.2 Foreign Debt in a Stationary Equilibrium. It is also the case that the amount borrowed by banks from the rest of the world is constant and non-negative in a stationary equilibrium, regardless of the type of foreign debt instrument and provided that there is no arbitrage of the interest rates. The same three cases for equilibria still apply, and thus, the structure of foreign debt of a bank in a steady-state equilibrium that belongs to Case j is given by the triplet $\left(\overline{d}_{0,j}, \overline{d}_{1,j}, \overline{d}_{2,j}\right) > 0$. We briefly describe these cases below.

Case 1: $j=1$ ***and no intra-date debt,*** where the vector $\left(\overline{d}_{0,1}, \overline{d}_{1,1}, \overline{d}_{2,1}\right) = \left(0, f_1 - f_0, f_0\right)$ obtains.

[35] Notice that we have switched the positions of τ_t and z_t to emphasize that all core dynamics originates from τ_t under a fixed exchange rate regime.

Case 2: $j=2$ ***and interior solution,*** with the vector $\left(\overline{d}_{0,2},\overline{d}_{1,2},\overline{d}_{2,2}\right)=\left(f_0-\overline{d}_{2,2},f_1-\overline{d}_{2,2},\overline{d}_{2,2}\right)\gg 0$. Specifically, the long-term debt in a steady-state equilibrium with a hard peg is given by

$$\overline{d_{2,2}}=\Omega_0\left(\sigma^*\right)+\left\{\left[\left(1+\sigma^*\right)\cdot r_2^*\cdot\varsigma_2\left(\sigma^*\right)+\left(\phi_d+\phi_f\right)\cdot\eta_2\left(\sigma^*\right)\right]/\left[r_1^*\cdot\left(1+\sigma^*\right)\cdot\left(r_0^*-r_1^*\right)\right]\right\}\cdot\lambda\cdot\overline{\tau}, \quad (26)$$

where the intercept $\Omega_0\left(\sigma^*\right)$ is defined in the following expression: $\Omega_0\left(\sigma^*\right)\equiv\left\{\lambda\cdot\left(1+\sigma^*\right)r_2^*\cdot\varsigma_1\left(\sigma^*\right)+\lambda\cdot\left(\phi_d+\phi_f\right)\left[\eta_1\left(\sigma^*\right)-w\right]-r_1^*\cdot\left(1+\sigma^*\right)\cdot\left(r_0^*-r_1^*\right)\right\}/\left[r_1^*\cdot\left(1+\sigma^*\right)\cdot\left(r_0^*-r_1^*\right)\right]$.

Case 3: $j=3$ ***and no long-term debt,*** where the vector $\left(\overline{d}_{0,3},\overline{d}_{1,3},\overline{d}_{2,3}\right)=\left(f_0,f_1-f_0,0\right)$ obtains.

As before, the stationary debt-structure vector permits us to calculate the current account balance in a stationary equilibrium that belongs to Case j:

$$\overline{CA_j}=\left(1-r_0^*\right)\cdot\overline{d}_{0,j}+\left(1-r_1^*\right)\cdot\overline{d}_{1,j}+\left(1-r_2^*\right)\cdot\overline{d}_{2,j}<0. \quad (27)$$

Notice that, under this policy as well, there is always a long-run deficit in the current account balance, which troubles the concept of long-run sustainability of this regime.

A stationary equilibrium under fixed exchange rates then is defined by the allocation $\left\{\left(\overline{\tau},\overline{z},\overline{q},\overline{b^*},\overline{k}\right),\left(\overline{d}_{0,j},\overline{d}_{1,j},\overline{d}_{2,j}\right),\left(\overline{c}_{1,j},\overline{c}_{2,j}\right)\middle|\ \overline{l}=0\right\}\in\mathbb{R}^5_{++}\times\mathbb{R}^3_{+}\times\mathbb{R}^2_{++}$, which satisfies all the conditions given above. The allocation of state-contingent commodities $\left(\overline{c}_{1,j},\overline{c}_{2,j}\right)\in\mathbb{R}^2_{++}$ follows directly from the debt-structure $\left(\overline{d}_{1,j},\overline{d}_{1,j},\overline{d}_{2,j}\right)$. Of course, the particular type of equilibrium and its properties will depend on the composition of the vector $\left(\overline{d}_{0,j},\overline{d}_{1,j},\overline{d}_{2,j}\right)$, as we will see below, as we check the conditions for existence, uniqueness and determinacy.

3.3.3 Existence and Local Uniqueness under a Fixed Exchange Rate Regime. In our model economy with a fixed exchange rate regime, domestic and foreign inflation are always equal, even in non-stationary allocations. In particular, in situations where the world inflation rate is high (low,) banks would have no (high) incentives to borrow long-term funds from abroad because inflation would undermine (boost) the real return of the currency reserves that banks must use when such debt is due[36]. Moreover, different combinations of foreign interest rates and world inflation may produce a variety of situations that translate into different equilibrium sets, since both foreign interest rates and world inflation represent costs that banks must face. Notice that high rates of world inflation may exacerbate the effect of the foreign interest rates, and thus, we focus on the role that σ^* plays in determining the existence of equilibria. The reader may notice some similarities in the proposition presented below with respect to existence under floating exchange rates, except for the fact the relevant parameter is σ^* instead of σ.

Proposition 5. Define the set $\mathbb{Q}=\left\{\sigma^*,r_0^*,r_1^*,r_2^*\right\}\in\mathbb{R}^4$ as the set of bifurcation parameters under a fixed exchange rate regime. Bifurcation values of these parameters partition $\mathbb{Q}$ into three subsets with defining characteristics that we describe below.

[36] Such combination of events could lead to a bank run and/or a panic among all agents, if there was uncertainty in this respect.

Subset 1= $\mathcal{Q}_1$ *:* Existence of Case 1 Equilibria. Given $\bar{\varepsilon}$ as defined in the Appendix, equilibria of Case 1 exist only when the following condition hold:

- Condition 1: $\hat{\varepsilon} > 0$ must hold[37].

Subset 2= $\mathcal{Q}_2$ *:* Existence of Case 2 Equilibria. This type of equilibrium always exists. Therefore, $\mathcal{P}_2 = \mathcal{P}$, and equilibria of Case 2 may coexist with equilibria of Cases 1 and 3.

Subset 3= $\mathcal{Q}_3$ *:* Existence of Case 3 Equilibria. Given the expressions $\bar{A}$, $\bar{B}$ and $\bar{C}$, as defined in the Appendix, equilibria of Case 3 exist when

- Condition 2: $\max\left\{\bar{A}, \bar{B}\right\} < r_1^* \leq \bar{C}$

Proof: This proof is very complex and lengthy. It is available upon request.

This second model economy with a hard peg in place violates as well the two standard conditions of regularity. With respect to the number of equilibria, we must mention that there is typically a continuum of equilibria in this economy. Moreover, regarding the mapping between the vectors of relative prices and the excess demand *correspondence* is not unique. Thus, steady-state equilibria in our economy are *"irregular,"* and thus, they are not locally unique nor they are determinate.

Allow us to elaborate more on the last statement. The core in the steady-state, given by the quintet $\left(\bar{\tau}, \bar{z}, \bar{q}, \bar{b}^*, \bar{k}\right)$, is always a unique and determinate vector, since it is not associated with the relative-price vector $\left(r_0^*, r_1^*, r_2^*\right)$. The problem, though, concerns the debt-structure vector, affecting through it the vector of state-contingent commodities as well. To begin with, given a fixed combination of parameters, each stationary debt-structure vector $\left(\bar{d}_{0,j}, \bar{d}_{1,j}, \bar{d}_{2,j}\right)$ is typically associated with a continuum of vectors of interest rates $\left(r_0^*, r_1^*, r_2^* = R\right)$, which are all consistent with the equilibrium conditions. Second, for a given price vector $\left(r_0^*, r_1^*, r_2^* = R\right)$, there may be more than one vector $\left(\bar{d}_{0,j}, \bar{d}_{1,j}, \bar{d}_{2,j}\right)$ that satisfies the equilibrium conditions. Thus, the nature of the absence of local uniqueness and determinacy is very similar to what we observe under floating exchange rates, saving of course different particulars and details such as the different role played by the world inflation rate in both regimes. The existence of the steady-state equilibria under both floating and fixed exchange rate regimes is strongly conditioned on the domestic policy parameter σ, for floating exchange rates, and the world inflation rate σ^* for a fixed exchange rate regime. In general, when one compare such two economies, $\sigma \neq \sigma^*$ would be typically observed. The latter implies that, a priori, the scopes for existence and indeterminacy are different, to some extent, under alternative exchange rate regimes. However, these properties cannot be compared strictly without additional information, making the task of ranking the alternative regimes in this respect somewhat difficult.

3.4 Dynamic Equilibria under a Fixed Exchange Rate Regime

Under a fixed exchange rate regime, the dynamics originates from the monetary authority's budget constraint, and the fact that both the money supply and the holdings of foreign-reserve assets must adjust to keep the

[37] Notice that $\mathcal{Q}_1$ is smaller than $\mathcal{P}_1$, reducing the likeliness of borrowing large amounts in long-term in equilibrium.

nominal exchange rate at its set level, $e > 0$. Next, we proceed by steps: we start with the core in dynamic equilibria, followed by foreign-debt dynamics, and we conclude with the dynamics of the state-contingent commodities.

3.4.1 The Core Dynamic System under a Hard Peg. The core dynamic system consists of the first order difference equations (22c), (22d), (23b), (23c) and (24a). Moreover, the dynamic system of the core is a de-coupled system where the dynamics is inherited from τ_t (see Figure 3.) We proceed by describing the dynamic behavior of each of the core variables, both with respect to σ^* and θ. Let $i = 1,2,3,4,5$ index each of the core variables $\left(\overline{\tau}_t, \overline{z}_t, \overline{q}_t, \overline{b}_t^*, \overline{k}_t\right)$, while $\left(\overline{\epsilon}_1, \overline{\epsilon}_2, \overline{\epsilon}_3, \overline{\epsilon}_4, \overline{\epsilon}_5\right)$ denotes the vector of associated eigenvalues. Proposition 6 below describes the dynamic properties of the core system.

Proposition 6 The dynamic properties of the core system depend strongly upon the world inflation rate (σ^*) and the fraction of the domestic supply (θ) backed with foreign-reserve assets by the monetary authority.
i) The World Inflation Rate. For each core variable i, where $i = 1,2,3,4,5$, the eigenvalue $\overline{\epsilon}_i(\sigma^*, \theta)$ is a monotonically increasing function of σ^*. In particular, $\forall i$, there exists a vector of the form $\left(\hat{\sigma}_i^*, \breve{\sigma}_i^*, \overline{\sigma}_i^*\right)$, such that $-1 < \hat{\sigma}_i^* < \breve{\sigma}_i^* < \overline{\sigma}_i^* < \infty$, $\overline{\epsilon}_i\left(\hat{\sigma}_i^*\right) = -1$, $\overline{\epsilon}_i\left(\breve{\sigma}_i^*, \theta\right) = 0$ and $\overline{\epsilon}_i\left(\overline{\sigma}_i^*, \theta\right) = 1$. Each entry in this vector is one of three bifurcation values that partition the support set of σ^* into four regions with defining characteristics. Specifically:

a) When $\sigma^* \in \left(-1, \hat{\sigma}_i^*\right)$, $\overline{\epsilon}_i\left(\sigma^*, \theta\right) < -1$ transpires, and unstable non-cyclical fluctuations are observed along dynamic paths around the stationary core.
b) For $\sigma^* \in \left(\hat{\sigma}_i^*, \breve{\sigma}_i^*\right)$, $\overline{\epsilon}_i\left(\sigma^*, \theta\right) \in (-1, 0)$ obtains, resulting in damped oscillations around the steady-state core.
c) $\overline{\epsilon}_i\left(\sigma^*, \theta\right) \in (0,1)$ occurs whenever $\sigma^* \in \left(\breve{\sigma}_i^*, \overline{\sigma}_i^*\right)$, causing stable monotonic dynamics around the stationary core.
d) For values of the world inflation rate that are sufficiently high (i.e. when $\sigma^* > \overline{\sigma}_i^*$ holds), $\overline{\epsilon}_i\left(\sigma^*\right) > 1$ ensues and dynamic paths diverge monotonically from the core steady-state.

ii) Backing of the Money Supply. Interesting dynamics and bifurcations are also observed for different values of $\theta \in [0,1]$. Each eigenvalue $\overline{\epsilon}_i\left(\sigma^*, \theta\right)$, for $i = 1,2,3,4,5$, is a monotonically increasing function of the policy parameter θ. Specifically, for each variable indexed by i that belongs to the core system, there exists a vector of bifurcation values $\left(\hat{\theta}_i, \breve{\theta}_i, \overline{\theta}_i\right)$ in the support set of θ such that $\hat{\theta}_i < \breve{\theta}_i < \overline{\theta}_i$, $\overline{\epsilon}_i\left(\sigma^*, \hat{\theta}_i\right) = -1$, $\overline{\epsilon}_i\left(\sigma^*, \breve{\theta}_i\right) = 0$ and $\overline{\epsilon}_i\left(\sigma^*, \overline{\theta}_i\right) = 1$ hold. Four situations may arise, and we describe them below.

a) $\hat{\theta}_i < 0 < \overline{\theta}_i < 1$: $\overline{\epsilon}_i\left(\sigma^*, \theta\right) < -1$ obtains, $\forall \theta \in [0,1]$, and, thus, no unstable fluctuations are observed. There is a scope for stable fluctuations, followed by monotonic stable dynamics and monotonic divergence as θ increases.
b) $0 < \hat{\theta}_i < \overline{\theta}_i < 1$: rules out monotonic divergence. For $\theta \in \left[0, \hat{\theta}_i\right)$, unstable fluctuations govern the dynamics around the stationary core. As θ increases, the oscillations gradually converge toward the steady-state core. Stable monotonic divergence ensues as θ continues to increase.
c) $\hat{\theta}_i < 0 < 1 < \overline{\theta}_i$: there is no diverging dynamics. Damped oscillations are observed along dynamic paths, giving place to stable monotonic dynamics as θ increases.
d) For $0 < \hat{\theta}_i < \overline{\theta}_i < 1$: the full spectrum dynamics comes about. Exploding oscillations take place for $\theta \in \left[0, \hat{\theta}_i\right)$, giving place to damped oscillations when $\theta \in \left[\hat{\theta}_i, \breve{\theta}_i\right)$. As θ continues to increase, stable monotonic dynamics ensue, followed by monotonic dynamics when $\theta > \overline{\theta}_i$.

3.4.2 The Dynamic System of the Debt-Structure under a Hard Peg. We now turn to discuss the composition and dynamic properties of the foreign-debt vector in dynamic equilibria that result from a fixed exchange rate regime. Below, we start our discussion of each case.

Foreign-Debt Dynamics in Cases 1 and 3: the equilibrium debt-structure in either of these extreme cases is stationary, as it was also the case under floating. With respect to Case 1, $\left(\overline{d_{0,1}}, \overline{d_{1,1}}, \overline{d_{2,1}}\right) = \left(\widehat{d_{0,1}}, \widehat{d_{1,2}}, \widehat{d_{2,1}}\right) = \left(0, f_1 - f_0, f_0\right), \forall t \geq 1$, and the corresponding current account $\overline{CA_1} = \widehat{CA_1} = \left(1 - r_1^*\right) \cdot f_1 + \left(r_1^* - r_2^*\right) \cdot f_0 < 0$ displays a nontrivial deficit. Concerning Case 3, foreign credit is given by $\left(\overline{d_{0,3}}, \overline{d_{1,3}}, \overline{d_{2,3}}\right) = \left(\widehat{d_{0,3}}, \widehat{d_{1,3}}, \widehat{d_{2,3}}\right) = \left(f_0, f_1 - f_0, 0\right), \forall t \geq 1$, while there is also a deficit in the associated current account balance $\overline{CA_3} = \widehat{CA_3} = \left(1 - r_1^*\right) \cdot f_1 + \left(r_0^* - r_1^*\right) \cdot f_0 < 0$.

Foreign-Debt Dynamics in Case 2: the debt-structure vector displays non trivial dynamics. As in Section 2, we present only the results for the long-term foreign debt instrument. The dynamics of $d_{2,t+1,2}$ under a fixed exchange rate regime is governed by

$$\overline{d}_{2,t+1,2} = \Omega_0\left(\sigma^*\right) + \Omega_1\left(\sigma^*\right) \cdot \overline{\tau}_{t-1}, \tag{28}$$

where $\Omega_1\left(\sigma^*\right) \equiv \lambda \cdot \left\{\left[\left(1+\sigma^*\right) \cdot r_2^* \cdot \varsigma_2\left(\sigma^*\right) + \left(\phi_d + \phi_f\right) \cdot \eta_2\left(\sigma^*\right)\right] / \left[r_1^* \cdot \left(1+\sigma^*\right) \cdot \left(r_0^* - r_1^*\right)\right]\right\}$. It is evident that (28) is a first order, linear difference equation in τ_t. The reader may notice that, so far, the dynamics under a fixed exchange rate is very different from what observed in the case of floating exchange rates.

<u>Local Stability Analysis.</u> The reduced-form coefficient $\Omega_1\left(\sigma^*\right)$ is the eigenvalue associated with the long-term debt, and it depends on the combination of parameters $\left(\sigma^*, \phi_d = \phi_f, \theta, \lambda\right)$ and the overall vector of returns $\left(\tilde{r}, r_0^*, r_1^*, r_2^*\right) \gg 1$. We want to point out that the structural parameter λ plays an important role in fixed-exchange-rate-dynamics, as opposed to the case with floating. Proposition 8 describes the *baseline sequence* as a function of the world inflation rate[38].

Proposition 7 Under a hard peg, the eigenvalue $\Omega_1\left(\sigma^*\right)$ is a monotonically increasing function of the world inflation rate. The *baseline sequence*[39] consists of a small range with unstable and very large non-cyclical fluctuations for values of σ^* sufficiently close to -1. As σ^* increases, the eigenvalue comes to lie inside the unit circle, with damped oscillations around the steady-state that turn gradually into stable monotonic dynamics when σ^* is sufficiently large.

In the Appendix, we evaluate how the latter *baseline sequence* changes with alternative combinations of parameters and returns.

3.4.3 The Dynamic System of the State-Contingent Commodities under a Hard Peg. Finally, we now turn to discuss the dynamic properties of the pair $\left(\overline{c}_{1,t,j}, \overline{c}_{2,t+1,j}\right) \gg 0$ in equilibrium with a fixed exchange rate regime in place. This dynamic system in equations (14c) and (24b) inherits its equilibrium laws of motion from

[38] Of course, domestic and world inflation rates are equal with a fixed exchange rate regime, and the latter is determined exogenously in the World Market for commodities.

[39] The definition of the *baseline scenario* with a hard peg is almost identical to the *baseline scenario* under floating. The only difference is that the parameter λ now plays an active role in the dynamics, and we set it to $\lambda = 0.2$.

the core system but mostly from the dynamic system of the foreign-debt structure. We start by discussing the dynamic properties of consumption by impatient agents in each of the different cases of equilibria. Next, we do the same for the consumption by patient agents.

Dynamics of the Consumption by Impatient Agents. The consumption by impatient agents is given by $\lambda \cdot \overline{c}_{1,t,j} = f_1 - f_0 - (r_0^* - 1) \cdot \overline{d}_{0,t,j}$, as it was the case under floating. In particular, for each case, we obtain

$$\lambda \cdot \overline{c}_{1,t,1} = f_1 - f_0, \text{ for } j = 1, \tag{29a}$$

$$\lambda \cdot \overline{c}_{1,t,2} = f_1 - r_0^* \cdot f_0 + (r_0^* - 1) \cdot \overline{d}_{2,t+1,2}, \text{ for } j = 2, \tag{29b}$$

$$\lambda \cdot \overline{c}_{1,t,3} = f_1 - r_0^* \cdot f_0, \text{ for } j = 3. \tag{29c}$$

We point out that $\overline{c}_{1,t,1} = \overline{c}_{1,1} = \hat{c}_{1,1}$ and $\overline{c}_{1,t,3} = \overline{c}_{1,3} = \hat{c}_{1,3}$, and thus, the consumption by patient individuals is always stationary in Cases 1 and 3 and identical to their counterparts under floating. However, in Case 2 the evolution of $c_{1,t,2}$ is governed by the term $(r_0^* - 1) \cdot \overline{d}_{2,t+1,2}$, indicating non-trivial dynamics. Specifically, the reduced-form dynamic equation for $\overline{c}_{1,t,2}$ is given by

$$\overline{c}_{1,t,2} = \Upsilon_0(\sigma^*) + \left[\Upsilon_1(\sigma^*) / (1 + \sigma^*) \cdot r_1^*\right] \cdot \overline{\tau}_{t-1}. \tag{30}$$

Notice that the coefficients in the last equation are given by $\Upsilon_0(\sigma^*) \equiv \{[(f_1 - r_0^* \cdot f_0) + (r_0^* - 1) \cdot \Omega_0(\sigma^*)] / \lambda\}$ and $\Upsilon_1(\sigma^*) \equiv (1 + \sigma^*) \cdot r_2^* \cdot \varsigma_2(\sigma^*) + (\phi_d + \phi_f) \cdot \eta_2(\sigma^*)$. The *baseline sequence* for $\overline{c}_{1,t,2}$ consists of a very small range of unstable fluctuations that turn very fast into damped oscillations as σ^* increases. For values of σ^* that are sufficiently high, there is a very large range of stable monotonic dynamics, which dominates the general dynamics. The minimum eigenvalue is extremely large and negative, indicating the presence of fairly large and unstable fluctuations for values of σ^* that are close enough to -1.

Dynamics of the Consumption by Patient Agents. The dynamic behavior of $\overline{c}_{2,t+1,j}$ is strongly influenced by $\overline{d}_{2,t+1,2}$, with some variations that depend upon the Case the equilibrium belongs to. This is apparent from comparing the following three first order linear difference equations:

$$\overline{c}_{2,t+1,1} = \Sigma_0(\sigma^*) + \left[(r_2^* - r_1^*) \cdot f_0 / (1 - \lambda)\right] + \left[\Upsilon_1(\sigma^*) / (1 - \lambda) \cdot (1 + \sigma^*)\right] \cdot \overline{\tau}_{t-1}, \tag{31a}$$

$$\overline{c}_{2,t+1,2} = \Sigma_0(\sigma^*) + \left[(r_2^* - r_1^*) \cdot \Omega_0(\sigma^*) / (1 - \lambda)\right] + \frac{\left[\lambda \cdot (r_2^* - r_1^*) + r_1^* \cdot (r_0^* - 1)\right]}{(1 - \lambda) \cdot (1 + \sigma^*) \cdot r_1^* \cdot (r_0^* - 1)} \cdot \Upsilon_1(\sigma^*) \cdot \overline{\tau}_{t-1}, \tag{31b}$$

$$\overline{c}_{2,t+1,3} = \Sigma_0(\sigma^*) + \left[\Upsilon_1(\sigma^*) / (1 - \lambda) \cdot (1 + \sigma^*)\right] \cdot \overline{\tau}_{t-1}. \tag{31c}$$

Notice that $(1 - \lambda) \cdot \Sigma_0(\sigma^*) \equiv r_2^* \cdot \varsigma_1(\sigma^*) + \left[(\phi_d + \phi_f) / (1 + \sigma^*)\right] \cdot \left[w + \eta_1(\sigma^*)\right] - r_1^* \cdot f_1$. From (31a) and (31c), it is straightforward that the eigenvalues of $\overline{c}_{2,t+1,1}$ and $\overline{c}_{2,t+1,3}$ are identical and, thus, these two variables share the same dynamic properties. All the eigenvalues in (31a)-(31c) share the same baseline sequence, with some minor variations. Proposition 9 summarizes our findings about the dynamic system of the state-contingent commodities.

Proposition 8 Let $m = 1,2,3,4$ index the variables $\left(\overline{c}_{1,t,2}, \overline{c}_{2,t+1,1}, \overline{c}_{2,t+1,2}, \overline{c}_{2,t+1,3}\right) \gg 0$ and the vector $\left(\overline{g}_1, \overline{g}_2, \overline{g}_3, \overline{g}_4\right)$ represent the associated vector of eigenvalues, where $\overline{g}_2 = \overline{g}_4$. All eigenvalues $\overline{g}_m\left(\sigma^*\right)$ are monotonically increasing functions of the world inflation rate, and, thus, all these variables display similar baseline sequences. The variations lie on the bifurcation values and the size of the minimum eigenvalue. The latter indicates the maximum size of the unstable fluctuations. The baseline sequence starts with a very small scope for unstable fluctuation when σ^* is very close to -1. Next, a small range with stable fluctuations arises as σ^* continues to increase, leading to a large and dominant scope for stable monotonic dynamics. The sizes of the unstable fluctuations are ranked as follows: $\min \overline{g}_1 < \min \overline{g}_2 = \min \overline{g}_4 < \min \overline{g}_3$. Thus, the consumption by patient agents in Case 2 exhibits the largest unstable fluctuations, while the consumption by impatient agents --also in Case 2, presents the smallest magnitude of diverging fluctuations. Finally, all the dynamic properties of $\overline{d}_{2,t+1,2}$ apply.

3.5 Comparing Equilibrium-Stability Properties under Alternative Exchange Rate Regimes.

We now summarize the results obtained along dynamic paths with a fixed exchange rate regime, and compare them against their equivalent under floating exchange rates. In the first place, all dynamic systems under a hard peg have first order difference equations, eliminating the possibility of cyclical fluctuations in the debt-structure vector that are typically associated with complex eigenvalues. Second, regarding the core, the full spectrum dynamics can be observed under fixed, while floating allowed only for monotonic dynamics; the latter implies that a fixed exchange rate regime promotes endogenously-arising volatility around the stationary core, while floating does not. There is a trade-off, however, vis-à-vis the foreign-debt structure and the state-contingent consumption: floating promotes higher order and very complicated dynamics that allow for nontrivial regions with complex eigenvalues in which cyclical and non-cyclical fluctuations are intertwined. In some cases, fluctuations can be significantly large and explosive, and the volatility may arise from a very large and unstable real part together with explosively-large and diverging amplitude of the cyclical component. A hard peg, instead, prevents the latter from occurring, and there is only a very small range for first order, simpler oscillating dynamics. Finally, the policy recommendations vary drastically across regimes: i) high and binding reserve requirements promote and extend the stability of dynamic equilibria under floating, while the only preserve stability and prevent monotonic divergence under fixed; ii) the backing of the money supply is a de-stabilizing policy parameter under floating, but it promotes stability under fixed; iii) the policy recommendations are exact opposites; floating requires very high reserve requirements and a very low backing of the money supply but an economy with fixed exchange regime is better-off with a combination of very low reserve requirements and a very high backing of the money supply.

4. Potential for Crises and the Vulnerability of Banks

In sections 2 and 3, we have pointed out that, in the absence of either extrinsic or intrinsic uncertainty, domestic agents will withdraw from banks according to their true types, and domestic banks anticipate this perfectly; of course, there will be no early liquidation the long-term asset. In addition, banks are liquid and solvent. At the beginning of date *t*, the domestic banks have chosen the amounts of state-contingent consumption vector $\left(c_{1,t}, c_{2,t+1}\right) \gg 0$ and had also formulated a plan that involved the core variables $\left(z_t, \tau_t, q_t, b_t^*, k_t\right)$, the debt-structure vector $\left(d_{0,t}, d_{1,t+1}, d_{2,t+1}\right) \geq 0$ and no early liquidation $\left(l_t = 0\right)$. Moreover, early morning in date t, banks and the monetary authority have made effective only their choices of the core variables, the currency reserves, $d_{0,t}$ $d_{2,t+1}$, and k_t. The variables $c_{1,t}$ and $d_{1,t+1}$ would be made effective at the end of date t, according to the initial contingent plan, while $c_{2,t+1}$ would be effective at the end of date $t+1$.

In this section, we focus instead on the case where an unanticipated shock hits the economy late in the afternoon of date *t*. This unexpected uncertainty may take one of two forms: a shock to the depositors' beliefs (i.e. a bad dream) or a sudden stop of foreign capital. In this section, the notation $\tilde{x}$ will indicate the re-optimized value of the variable x in the sense that it deviates from the original contingent plan formulated by banks in the absence of a shock.

4.1 Sources of Uncertainty

Extrinsic Uncertainty is a type of uncertainty that typically exacerbates the beliefs of the general public –the depositors in our case—without being associated with any change in the fundamentals of the economy. It is frequently linked with outcomes of a self-fulfilling type: a prophecy that is realized as an outcome through the beliefs and actions of the main economic actors. The literature frequently illustrates the properties of this type of shock by using the example of the young domestic depositors having a "*bad dream*" in which banks will close, but without any change to the fundamentals of the economy. Interestingly, we observe complementarity in the strategic interaction between the individual depositors: if an individual agent cannot observe the actions or types of others, she may panic if she believes that everyone in the economy has had the same bad dream. In other words, if she believes that everybody else is going to run and banks have a sequential service constraint, she will run as well. Whether depositors run or not will determined by the results obtained from the sequential checking mechanism in Figure 4. In this case, banks do not reoptimize since there is no change in the fundamentals. Then, in the aggregate, all domestic depositors may run on the banks to withdraw their resources immediately with the purpose of getting there before any other individual in the economy does, and we would have a self-fulfilling bank-run in our hands. In our framework, the realization of this sunspot variable typically takes place at the end of period *t*, after the individuals learn their types. In a particular set of circumstances, this "*bad dream*" has the potential of greatly affecting the outcomes in the economy by leading to panic-equilibria, which display significantly lower welfare.

Intrinsic uncertainty is another type of uncertainty. It has to do with an unanticipated change in one or more of the fundamentals of the economy after agents made their decisions. A sudden stop certainly belongs in this category. In this paper, we replicate a sudden stop as an unanticipated and exogenous reduction of the amount of new credit available from the rest of the world ($f_1 > 0$) that takes place before the end of the first date of the individuals' lives --date t. Recall that the pair $\left\{f_0, f_1\right\}$ was set exogenously in the rest of the world at the beginning of date t, and that contingent choices were made at that point. The two constraints on foreign credit were binding and banks had already acted on their choices of $d_{0,t}$ and $d_{2,t+1}$, but their choices of $d_{1,t+1}$ and l_t have not been made effective yet. When a sudden stop hits the economy, it abruptly reduce the resources to f_1' goods, where $0 \leq f_1' < f_1$ obtains. The relevant foreign credit constraint now becomes

$$d_{2,t+1} + \tilde{d}_{1,t+1} = f_1' \quad (32)$$

We will see later that this intrinsic shock may reduce both liquidity and solvency in the financial sector[40], thus having the potential of triggering a domestic crisis that may take the form of generalized bank runs, bankruptcy and closure in the domestic financial system.

Once such a shock hits the economy, this event becomes public information and the availability of future resources changes irreversibly. Under this new set of circumstances, it may be in the depositors' best interest to withdraw from banks as much as possible immediately, and the presence of a sequential service constraint may only exacerbate this problem. In these circumstances, it may not be optimal for any agent – specially, agents of the patient type-- to wait until the next date to withdraw from the banks, since they believe that banks could be facing bankruptcy and closure. Both banks and depositors will need to re-optimize to account for the change in circumstances, leading to what we call a Sequential Checking Mechanism in Figure 4. This mechanism consists of three steps. The first step is an evaluation of the liquidity position of banks. Depending on the results obtained in the first step, evaluating the banks' solvency becomes the next priority, followed by checking whether the resulting allocations are incentive-compatible or not.

4.2 Sequential Checking Mechanism

Liquidity. Chang and Velasco (2000a, 2000b and 2001) were among the first in the literature to mention the need to evaluate the liquidity position of banks in the context of financial crises in the emerging markets. They propose the use of an illiquidity condition that summarizes the current situation of banks after a shock hits the domestic economy at the end of date *t*, assuming that a bank cannot commit *not* to liquidate fully the long-term investment if needed. We now describe this condition.

On the one hand, if all the depositors decided to withdraw their deposits, the real value of the bank's short-term obligations due at the end of date *t* would be $c_{1,t} + r_0^* \cdot d_{0,t}$ goods. On the other hand, if the bank were to liquidate early the full amount of its long-term investment at the end of time *t*, $\tilde{l}_t = k_t$ and it would obtain $r \cdot \tilde{l}_t = r \cdot k_t$ goods from this action[41]. Moreover, the illiquidity condition assumes no additional funds are available at this point in time (i.e. no bailing out). In summary, one says that the representative bank has an

[40] We could also argue that unanticipated reductions in foreign credit may trigger a shock to the preferences of depositors. If such a shock induces a crisis of a self-fulfilling nature, this may only exacerbate the existing problems in this economy. In this paper, for simplicity, we abstract from this possibility.

[41] Recall than, in the absence of either extrinsic or intrinsic uncertainty, there was no early liquidation in equilibrium, and thus $l_t = 0$ obtained. When we allow for either one of these two types of uncertainty, and a shock is realized, individuals may re-optimize, and the new choice of early liquidation would be $\tilde{l}_t \geq l_t = 0$.

illiquid position when the real value of its short-term obligations at the end of date *t* exceeds the liquidation value of the long-term investment, or equivalently, when the following inequality holds:

$$c_{1,t} + r_0^* \cdot d_{0,t} > r \cdot k_t \,. \tag{33}$$

Of course, when the inequality in (33) does not obtain, we say that the bank has a liquid position.

Solvency. One of our main points is that an illiquid position is a necessary but not sufficient condition for a bank-run equilibrium (bankruptcy) to obtain in our model. The short-term resources of the bank may be insufficient at the end of date *t*, but the bank could still be solvent next date if it is temporarily bailed-out. The banks' illiquidity may only be a temporary matter caused by the shock that could be solved if foreign lenders would provide them with a provisional bail-out in the amount of $\tilde{d}_{1,t+1}$ goods. The rule is to bail out solvent banks, but let the insolvent ones close. We can summarize this idea with the following saying: *"why throw away good money after bad money?"*

When discussing the issue of solvency, the resources available include not only $r{\cdot}k_t$ but also the real value of foreign debt newly chosen at *t*, $0 < \tilde{d}_{1t+1} \leq f_1'$. We say that the representative bank is solvent if it still has enough resources left after covering both the withdrawals of deposits and payments to foreign intra-date debt, even if the bank were to liquidate early the total of its long-term investment. Equivalently, the following inequality describes the conditions for the insolvency of a representative bank at the end of date *t*:

$$c_{1t} + r_0^* \cdot d_{0t} > r \cdot k_{t+1} + \tilde{d}_{1t+1} \tag{34}$$

The inequality in (34) means that if the real value of the new short-term foreign debt $\tilde{d}_{1t+1}$ (i.e., the bail out) is enough to alleviate the temporary problem of liquidity, and that it would be in the best interest of the foreign creditors to help this bank out instead of forcing it to close, so that they can still recover at least the amount $d_{2,t+1}$ that they lent long-term to domestic banks at the beginning of date t.

Incentive Compatibility. In a situation where banks are illiquid and solvent, it is still possible that a fraction of the patient agents may still have incentives to misrepresent their type and withdraw early, leading to panics and closure. Thus, we incorporate the need to evaluate whether the Incentive Compatibility constraint in (2) is satisfied or not. On the other hand, if, in spite of the bail-out, the banks are still insolvent, it will not be bailed-out in equilibrium. The latter will be public information and depositors[42] will run on banks and withdraw all of the deposits, leading to the bankruptcy and closure of banks.

4.3 Type of Equilibria and Re-optimization

[42] This may sound familiar to the reader: as of September of 2008, the Federal authority has evaluated investment banks using a similar criterion, given the recent events subsequent to the subprime mortgage crisis. However, the origins of the aforementioned crisis are different from a sudden stop.

After one of the aforementioned shocks hits our model economy, banks formulate a new plan. If the chock realized takes the form of extrinsic uncertainty, then $\left(\tilde{d}_{1,t+1}, \tilde{l}_t\right) \neq \left(d_{1,t+1}, l_t\right)$. However, if intrinsic uncertainty is in place, $\left(\tilde{d}_1, \tilde{l}\right) \neq \left(d_1, l\right)$, one would typically expect that $\tilde{d}_{1,t+1} < d_{1,t+1}$ and $\tilde{l}_t > l_t$ under the circumstances. Following the simple logic behind these three conditions with an example, we were able to differentiate four different sets in which equilibria may exist:

a) Equilibria of Type 1: Banks have a liquid position. Under these circumstances, banks are liquid and solvent, and there are no panics. We will call a situation like this a *separating non-panic equilibrium where banks hold liquid positions*.

b) Equilibria of Type 2: Banks have an illiquid position, but they are solvent and the Incentive Compatibility constraint is satisfied. Under this set of circumstances, the bank is illiquid but no panics occur. We will consider such an outcome a *separating non-panic equilibrium where banks hold illiquid positions*.

c) Equilibria of Type 3: Banks have an illiquid position, they are solvent, but the Incentive Compatibility constraint is not satisfied. Then, banks are illiquid and solvent but there are panics in equilibrium, and all banks must close. We will call this outcome a *pooling equilibrium with panics but solvent banks.*

d) Equilibria of Type 4: Banks have an illiquid position and they are insolvent. Panics occur in equilibrium, and all banks must close. This situation is a *pooling equilibrium with panics, illiquid and insolvent banks.*

In summary, equilibria of Types 1 and 2 are good separating equilibria where depositors behave according to their true type. Panics do not occur in good separating equilibria. However, equilibria of Types 3 and 4 are pooling equilibria where panics do occur. Proposition 10 below presents a ranking of the 4 types of equilibria, based on social welfare.

Proposition 10 The following welfare ranking obtains:

i. Equilibria of Type 1 Pareto dominate equilibria of Type 2, Type 3 and Type 4.
ii. Equilibria of Type 2 are Pareto superior to equilibria of Type 3.
iii. Equilibria of Type 3 Pareto dominate equilibria of Type 4.

Proof: available upon request.

A Sudden Stop and the Re-Optimization Problem At the beginning of date *t*, banks choose their plan of action based upon, among other things, the pair $\{f_0, f_1\}$. Banks formulate their plans by maximizing the *ex-ante* expected utility of a representative individual, subject to the relevant constraints. At the end of date *t*, banks learn the realization of $f_1' \in [0, f_1)$. This realization constitutes an irreversible reduction in the future resources available to banks that merits a revision of plans. Under the new credit constraint (32), it may be possible that some decision rules that were optimal before are not optimal anymore. However, at this point in time, only two choice variables remain elastic: d_{1t+1} and l_t, since $d_{0t}, d_{2t+1}, k_{t+1}$ and the reserve-holdings were chosen and made effective at the beginning of *t* (and are, thus, inelastic). Banks may choose to deviate from their original plans. They would do so by finding a new combination $(\tilde{d}_{1t+1}, \tilde{l}_t)$ that is optimal under the new circumstances.

After the sudden stop hits the domestic financial sector, this news is public. The sequence of events that follows the shock are everybody realizes that the amount of new borrowing from abroad is less than it was when they formulated their original plans. Fortunately, people can still try to behave optimally by sequentially checking the situation of the banks. Thus, the need to re-optimize and re-formulate their plans triggers a new sequential checking to try to determine the best course of action under the new circumstances, and equilibria obtain accordingly.

The new borrowing constraint faced by banks at the end of period *t* implies that

$$\tilde{d}_{1t+1} \leq f_1' - d_{2t+1}. \tag{35}$$

The sequential checking mechanism now requires re-evaluating (33), (34) and (2) by taking into account (35) as well. Banks now try to maximize the expected utility in (1) subject to the new budget constraint in (35), and the relevant budget constraints according to the exchange rate regime.

4.4 Overview of Equilibrium Results-Floating Exchange Rate

Henceforth, we focus on stationary allocations where $f_1' > f_0$. We now describe the new equilibria that result after re-optimization and the sequential checking process. At the time of the shock, the variables d_0, d_2 and k are already realized. Similarly, the pair (c_1, c_2) is already determined by the deposit contract. Thus, d_0, d_2, c_1, c_2 will not change as a consequence of the shock.

However, banks must have to change d_1 to $\tilde{d}_1$ and l from zero to $\tilde{l} > 0$. To make these changes effectually, banks will first set $\tilde{d}_1 = f_1' - d_2 < d_1$, and next will solve for $\tilde{l}$ using equation (6). Thus, the new early liquidation of the long-term investment is given by

$$\tilde{l}_t = f_0 + (1 - \phi_d - \phi_f)(w + \tau_t) + \frac{\phi_d (w + \tau_t)}{R(1+\sigma)} + \frac{\phi_f (w + \tau_t)}{R(1+\sigma^*)} - \frac{(r_2^* - r_1^*) d_2}{R} - \frac{r_1^* f_1'}{R} \tag{36}$$

We must point out a couple of things. First, $\tilde{l}$ is monotonically decreasing in f_1', which ensures that $\tilde{l} > 0$. Second, $\tilde{l}$ is also a monotonically decreasing function of d_2. It follows then, that $\tilde{l}$ will vary with the potential values of d_2. We can now partition the set of possible types of equilibria according to different sets of values of the foreign debt triplet $(d_0, \tilde{d}_1, d_2)$. It follows that the set of equilibria can partitioned into three subsets:

- <u>Subset 1</u>: $(d_0, \tilde{d}_1, d_2) = (0, f_1' - f_0, f_0)$
- <u>Subset 2</u>: $(d_0, \tilde{d}_1, d_2) = (f_0 - d_2, f_1' - d_2, d_2)$

- Subset 3: $\left(d_0, \tilde{d}_1, d_2\right) = \left(f_0, f_1^{'}, 0\right)$

We now define $\tilde{l}_j$, for j=1,2,3, to be the early liquidation in equilibria of Type j. it is straightforward that $\tilde{l}_1 < \tilde{l}_2 < \tilde{l}_3$.

When the sudden stop of foreign credit hits the economy, the anxious domestic depositors and foreign creditors start checking the bank's capacity of operation. Under the illiquid situation, credit crunch among foreign creditors will directly impact the bank's solvency. We now define the functions A_j, B_j, and C_j associated with the type of equilibria j discussed. A_j corresponds to the boundary condition of f_1 for illiquidity in Sub-set j, B_j represents the boundary condition of $f_1^{'}$ for solvency in Sub-set j, and C_j denotes the boundary condition of f_1 for incentive compatibility in Sub-set j [43]. We summarize the results in Table 4.

In particular, in order to determine the type of equilibrium, we must find the appropriate regions in the space of foreign interest rates and monetary policy parameters where liquidity, solvency and self-selection intersect. We have pointed out the Equilibria of Type 4 yield the lowest social welfare possible –even worse than autarky, and they can be considered an outcome that the society as a whole will be willing to prevent. Only when the bank is illiquid and lacks solvency at the same time, the bank will close and claim a bankruptcy. Otherwise, panic equilibria do not exist when the patient depositors choose to wait. Interestingly, equilibria of Type 4 do not exist in the Subsets 1 and 3 but they do in Subset 2, indicating the potential for financial fragility and severe welfare losses that are latent in the two model economies, even in the presence of binding reserve requirements.

In Subset 1, the bank borrows a relatively large amount of foreign long-term debt (d_2) at the beginning of period t, restricting the bank's fund-raising ability in the future: after the sudden stop of foreign credit, the access to the new short-term funds becomes even scarcer. Nevertheless, at time t $d_0 = 0$ and there are no foreign obligations to repay; the only liability the bank has to deal with is the withdrawals from the impatient depositors (c_1) if there is no panic in the economy. Hence, bank has difficulty to finance its obligations through short-term borrowing ($\tilde{d}_1$), and illiquidity will force the bank to liquidate its high-earning investment to fulfill the need of depositor's withdrawals. But, in Subset 1, banks are never insolvent and equilibria of Type 4 do not obtain.

We now turn to explain another channel of effects that may seem counterintuitive. Because in Subset 1, $\tilde{d}_1$ is very small and $d_0 = 0$, then the early liquidation of the long-term investment need not be large. On the country, banks will expect a higher return at the end of t+1. Given the latter, we will observe that for larger values of $\tilde{d}_1$, banks will have to liquidate a larger part of their long-

[43] The detail of each function can be found in Appendix 3.

term investment, increasing the likelihood of moving toward equilibria of Type 2 and even Type 3.

Regarding equilibria in Subset 2, there is a scope for allocations that are not incentive compatible or display insolvent banks. Thus, equilibria of Type 4 may obtain, accompanied by very low levels of social welfare. Even through foreign lending could serve as the last source for the illiquid bank, depositors beliefs may deteriorate the economy into a run. Equilibria of Type 3 exist since incentive compatibility is violated for particular values of the monetary policy parameter σ and the foreign interest rates.

Unlike Subset 1, in Subset 3 banks take out no long-term loans from abroad at time *t*. That means the bank will have greater flexibility for short-term borrowing if needed. It also implies that the total amount it has to repay at time *t*+1 is relatively manageable. If there is a shock and it permanently reduces the sources of funds to the bank, this bank has the ability to gather sufficient amount of bail-out funds from foreign creditors. The later eliminates the likelihood of observing equilibria of Type 4.

To sum up, the economy may pivot from Type 1 (with the highest social welfare) to Type 4 (with the lowest social welfare) due to infinitesimal changes in f_1' and/or in policy parameters, illustrating a very fragile and highly volatile environment faced by the financial system, which could lead to panics and generalized bankruptcy.

4.5 Evaluation of the Role of the Policy

Our model provides a suitable framework for analyzing the interaction among various types of monetary policies: controlling the domestic rate of money growth, inflation targeting, fixed versus floating exchange rate regimes and regulations of multiple reserve requirements.

Reserve Requirements. In this section we investigate the role of a policy of reserve requirements in changing the scope for existence and determinacy of different types of equilibria. We illustrate our findings in Table 5. The main purpose of multiple reserve requirements is to reduce the likelihood of panics and bankruptcy. Under this policy rule, all banks must put aside a fraction of their deposits in the form of currency reserves at the beginning of date t, before allocating their investment portfolios. These currency reserves are returned to banks only at the end of date $t+1$, time at which banks repay all foreign debts and the remaining withdrawals from depositors. They provide banks with extra-liquidity at the end of $t+1$, thus liberating some resources for banks to choose how much of the long-term asset to liquidate at the end of date t. Depositors are less likely to panic in the face of a shock.

Let us turn now to the practical effects of reserve requirements observed in our model economies when reserve requirements are set to zero. First, the existence of the equilibria of type 1 and 2 reduces in Subset 1 and 3 when $\phi_d = \phi_f = 0$. If the bank liquidates all its long-term investment early right after the sudden stop of foreign credits, without the back of the reserves, there is no resource left for this bank to finance the obligations in the period *t*+1. On the other hand, when the bank finds its way out of the insolvency by maximizing the foreign borrowing, without required reserves, the maturity of the new short-term debts increases the burden of the bank and may drive the patient depositors to run before the deposit contract is matured. Therefore, when $\phi_d = \phi_f = 0$, the Panic Type 3 equilibria are more likely to happen and the chance for the good equilibria to exist will be reduced.

In the especial Sub-case 2, eliminating reserve requirements has no clear effects on the range of two extreme equilibria of Type 1 and 4, but the ranges for Non-panic equilibria of 2 are reduced significantly while the range for Panic equilibria of Type 3 increases. Thus, in the absence of reserve requirements, the economy moves away from two polar points. From table 5, the economy will gravitate more towards equilibria of Type 3 than to equilibria of Type 2. That means the financial system can be fragile and volatile and further leading to financial panic and closure.

One can then argue that reserve requirements play a stabilizing role in our model economies in the following ways: 1) They reduce the polarization of economic outcomes by providing the economy with some intermediate and less tenebrous kinds of equilibria; 2) They increase the scope for the economy to attain highest welfare equilibria (Type 1;) 3) They reduce significantly the likelihood of the economy facing the worst possible outcomes (Type 4;) and 4) They significantly reduce the vulnerability and volatility of the economic environment. All this takes place regardless of the exchange rate regime in place.

Corresponding Monetary Policy. In this section, we want to know how monetary policy affects the structure of the economy. When there is a new monetary policy in place or there is an unexpected increase in domestic money growth, the range for equilibria to exist varies drastically depending on which way the bank is going to finance its unexpected withdrawals. The equilibria results are summarized in Table 6.

The society that has a higher money growth tends to achieve Type 1 equilibria and prevent Type 4 equilibria when the bank chooses a proper combination to balance the new foreign borrowing against the early liquidation. On the other hand, if the bank goes to the two bipolar extremes, financing all through either liquidating or all through new foreign borrowing, under the circumstance with a relatively higher inflation, it has a greater chance to achieve equilibria of Type 3. The equilibria with higher welfare, equilibria of Type 1 and Type 2, are not obtainable.

The explanation behind this result is following. A high money growth implies a greater possibility of inflation, which deteriorate the real value of the reserve requirements. Therefore, the economy tends to move towards the good equilibria when monetary policy and the channel of foreign borrowing reduce the panic among depositors. The situation changes dramatically if the bank found it hard to finance from abroad. Domestic financial intermediaries experiencing credit crunch will find the external funds are too far to reach in the shortage of the credit market. Thus, the bank that has to liquidate its investment early in order to become solvency at time *t* may moves towards a bank run at time *t*+1. Therefore, under a higher money growth, patient depositors are more likely to cheat and misrepresent their types.

To sum up, a loosened monetary policy affects the structure of our model economies in the following ways: 1) It reduce significantly the likelihood of the economy facing the worst possible outcomes (Type 4) if the bank has a channel to borrow after the shock; 2) It increases the likelihood of the Panic Type 3 equilibria in all Sub-cases; 3) The effect on Type 1 and 2 equilibria is unclear and will depend on the amount of new foreign borrowing.

5. Conclusions

In this paper, we investigate whether a particular monetary policy can help prevent financial crises that originate in illiquid positions by banks. We then compare the advantages and disadvantages of alternative *de facto* exchange rate regimes in achieving economy stability. Finally, we evaluate the impact that a policy of multiple reserve requirements can have on the scope for existence of equilibria that are not vulnerable to crises.

In order to call attention to the questions that we are interested in, we built a dynamic general equilibrium model of a small, open economy that displays nontrivial demands for fiat currencies, unexpected sunspots, and financial/banking crises originated by sudden stops of foreign capital inflows. We motivated the effective demands for domestic and foreign fiat money with a policy of holding reserves of different national currencies. These reserves prevent banks from financing domestic investment, but they may also provide banks with access to liquid resources in their time of need. Under some particular circumstances, reserve requirements may reduce the likelihood of observing equilibria with illiquid and insolvent banks.

In situations where no crises are present, we observe that the monetary rule in place determines up to an important extent the existence and properties of equilibria. Typically, there is a continuum of stationary equilibria, and local uniqueness and determinacy are lacking. With respect to dynamic equilibria under floating exchange rates, there is a nontrivial scope for complex eigenvalues that contributes to both cyclical and non-cyclical fluctuations; in some cases, the fluctuations can be significantly large and explosive. The scope for stability --and indeterminacy-- is typically small, and the scope for determinacy typically dominates, but fluctuations are observed on the stable manifold. Moreover, unstable and oscillating divergence is observed in general. The reserve requirements play the role of stabilizing, at least partially, the dynamic equilibria in this model economy, while backing the domestic supply plays the opposite role. Thus, these results provide us with policy recommendations: to implement high and binding reserve requirements, but keeping the backing of the money supply to a minimum. In the extreme case of $\theta = 0$, the order of the dynamic system is reduced, which one can interpret as the ultimate stabilization of dynamic equilibria.

With respect to dynamic paths under a hard peg, we must point out following. In the first place, all dynamic systems under a hard peg have first order difference equations, eliminating the possibility of cyclical fluctuations in the debt-structure vector that are typically associated with complex eigenvalues. Second, regarding the core, the full spectrum dynamics can be observed under fixed, while floating allowed only for monotonic dynamics; the latter implies that a fixed exchange rate regime promotes endogenously-arising volatility around the stationary core, while floating does not. There is a trade-off, however, vis-à-vis the foreign-debt structure and the state-contingent consumption: floating promotes higher order and very complicated dynamics that allow for nontrivial regions with complex eigenvalues in which cyclical and non-cyclical fluctuations are intertwined. In some cases, fluctuations can be significantly large and explosive, and the volatility may arise from a very large and unstable real part together with explosively-large and diverging amplitude of the cyclical component. A hard peg, instead, prevents the latter from occurring, and there is only a very small range for first order, simpler oscillating dynamics. Finally, the policy recommendations vary drastically across regimes: i) high and binding reserve requirements promote and extend the stability of dynamic equilibria under floating, while the only preserve stability and prevent monotonic divergence under fixed; ii) the backing of the money supply is a de-stabilizing policy parameter under floating, but it promotes stability under fixed; iii) the policy recommendations are exact opposites; floating requires very high reserve requirements and a very low backing of the money supply but an economy with fixed exchange regime is better-off with a combination of very low reserve requirements and a very high backing of the money supply.

Next, we examined the case of a sudden stop in a small, open economy that is a net borrower from the rest of the world. We show the existence of multiple equilibria that we may rank based on the presence of binding information constraints and on social welfare, where bank-runs with insolvent banks is the least-preferred outcome. Not surprisingly, we also find that the magnitude of the reduction in foreign credit determines the existence of different types of equilibria and their likelihood to occur. However, more importantly, we find that a policy of reserve requirements reduces the polarization of economic outcomes by providing the economy with some intermediate and less tenebrous kinds of equilibria, it increases the scope for the economy to attain highest welfare equilibria, and it reduces significantly the likelihood of the economy facing the worst possible outcomes. Finally, this policy may reduce the scope for fragility and volatility of the banking system, regardless of the exchange rate regime in place. Future research: money supply, liquidity preference, and domestic interest rate under floating.

Table 1.A: Evolution of Exchange Rate Regimes in the East Asian Countries

Country	Period		Exchange Rate Regime Classification	
	From	To	Narrow Classification	Broad Classification
Hong Kong	Jul-72	Oct-74	Peg to U.S. dollar	Fixed
	Nov-74	Oct-83	Independently floating	Floating
	Oct-83	Dec-04	Peg to U.S. dollar	Fixed
Indonesia[a/]	Jan-83	Jul-98	Managed floating	Intermediate
	Aug-98	Jan-02	Independently floating	Floating
Japan	Jan-70	Dec-72	Bretton Woods basket peg	Fixed
	Jan-73	Dec-04	Independently floating	Floating
Korea, Rep. of	Aug-76	Dec-79	Peg to U.S. dollar	Fixed
	Jan-80	Nov-97	Managed floating	Intermediate
	Dec-97	Dec-04	Independently floating	Floating
Malaysia	Jan-70	Jun-72	Peg to pound sterling	Fixed
	Jul-72	Jun-73	Peg to U.S. dollar	Fixed
	Jul-73	Aug-75	Independently floating	Floating
	Sep-75	Mar-93	Limited flexibility w.r.t. a basket	Intermediate
	Apr-93	Aug-98	Managed floating	Intermediate
	Sep-98	Dec-04	Peg to U.S. dollar	Fixed
Philippines	Oct-81	Jun-82	Limited flexibility w.r.t. U.S. dollar	Intermediate
	Jul-82	Sep-84	Managed floating	Intermediate
	Oct-84	Dec-04	Independently floating	Floating
Singapore	Aug-73	Jun-87	Limited flexibility w.r.t. a basket	Intermediate
	Jul-87	Dec-04	Managed floating	Intermediate
Thailand [a/]	Jan-77	Feb-78	Peg to U.S. dollar	Fixed
	Mar-78	Jun-81	Limited flexibility w.r.t. a basket	Intermediate
	Jul-81	Mar-82	Managed floating	Intermediate
	Apr-82	Oct-84	Limited flexibility w.r.t. U.S. dollar	Intermediate
	Nov-84	Jun-97	Limited flexibility w.r.t. a basket	Intermediate
	Jul-97	Jun-98	Managed floating	Intermediate
	Jul-98	Jan-02	Independently floating	Floating

[a/] The information from the Central Bank of Indonesia (floating) is different from that of the IMF (managed floating) after the year 2002. The same happens in Thailand. *Source: Frankel et al (2002).*

Table 1.B: Exchange Rate Regimes in the East-Asian Countries Before the Crisis and After the Crisis

Country	Before/During the crisis	After the crisis
Japan	Free floating	Free floating
Philippines	Free floating	Free floating
China	Managed floating	Managed floating
Indonesia	Managed floating	Floating
Korea	Managed floating	Floating
Singapore	Managed floating	Managed floating
Thailand	Managed floating	Managed floating $\rightarrow$ floating
Malaysia	Managed floating	Fixed
Hong Kong	Fixed	Fixed

Source: Frankel et al (2002)

Table 2: Existence of Steady-State Equilibria under Floating Exchange Rates, $\hat{l} = 0$

Case	**Debt-structure**	**Existence**	**Scope for existence and σ**
Case 1	$d_2 = f_0$ $d_0 = 0$ $d_1 = f_1 - f_0$	1) $\hat{\varepsilon} > 0$, or 2) $\sigma < \dfrac{\hat{\varepsilon} + \lambda \cdot \phi_d \cdot w}{-\hat{\varepsilon}}$	1) Increases with σ when ε>0 and $\lambda \cdot w > r_1^* \cdot (f_1 - f_0)$ holds.
Case 2	$0 < d_2 < f_0$ $d_0 = f_0 - d_2$ $d_1 = f_1 - d_2$	Equilibria always exist	Unchanged with changes in σ
Case 3	$d_2 = 0$ $d_0 = f_0$ $d_1 = f_1$	$\max\{A, B\} < r_1^* \leq C$ must hold	1) When $B < A < r_1^* \leq C$ holds, it increases with σ. 2) When $A < B < r_1^* \leq C$ holds, it increases with σ only when $\left(\dfrac{2 \cdot \lambda \cdot f_1}{f_1 - r_0^* \cdot f_0}\right) < \hat{\Omega}$ holds [44].

[44] Where $\hat{\Omega} \equiv \left| 1 - \dfrac{(1-\lambda) \cdot r \cdot f_1 - \lambda \cdot R \cdot f_0 - \bar{a}}{\left\{ \left[(1-\lambda) \cdot r \cdot f_1 - \lambda \cdot R \cdot f_0 - \bar{a} \right]^2 + 4 \cdot \lambda \cdot (1-\lambda) \cdot r \cdot R \cdot f \cdot_1 f_0 \right\}^{1/2}} \right|$.

Table 3: Dynamic Properties of the Real Balances of *wons* z_t - Floating Exchange Rates

θ	ϕ_d	σ	$\alpha_2(\sigma)$
0	$(0,1)$	> -1	0
$(0,1)$	0	> -1	0
$(0,1)$	$(0,1)$	$< \tilde{\sigma}$	< 1
$< \hat{\theta}$	$< \hat{\phi}_d$	$\geq \tilde{\sigma}$	< 1
$\geq \hat{\theta}$	$\geq \hat{\phi}_d$	$\geq \tilde{\sigma}$	≥ 1
$(0,1)$	1	$< \tilde{\sigma}$	< 1
$< \hat{\theta}$	1	$\geq \tilde{\sigma}$	< 1
$\geq \hat{\theta}$	1	$\geq \tilde{\sigma}$	≥ 1
$(0,1)$	$< (1/\tilde{r})$	∞	< 1
$< \hat{\theta}$	$\geq (1/\tilde{r})$	∞	< 1
$\geq \hat{\theta}$	$\geq (1/\tilde{r})$	∞	≥ 1
$(0,1)$	1	∞	≥ 1
1	1	∞	∞

Note: $\hat{\theta} \equiv [(\tilde{r}-1)\cdot(1+\sigma)]^{-1}$, $\hat{\phi}_d \equiv (1+\sigma)\cdot\{\theta\cdot(\tilde{r}-1)+\sigma\cdot[\theta\cdot(\tilde{r}-1)+1]\}^{-1}$, and $\tilde{\sigma} \equiv (2-\tilde{r})/(\tilde{r}-1)$.

Table 4: Existence of Steady-State Equilibria after Shock

Sub-set		Range for ***Type 1***	Range for ***Type2***	Range for ***Type 3***	Range for ***Type 4***
1.	$\tilde{d}_{1t+1} = f_1^{'} - f_0$	$B_1 < f_1^{'} < f_1 < A_1$	$B_1 < A_1 < f_1^{'} < f_1 < C_1$	$B_1 < A_1 < C_1 < f_1^{'} < f_1$	Never Exist
2.	$\tilde{d}_{1t+1} = f_1^{'} - d_{2t+1}$	$f_1^{'} < f_1 < A_2 < B_2$	$B_2 < f_1^{'} < f_1 < C_2$	$B_2 < f_1^{'} < C_2 < f_1$ or $B_2 < C_2 < f_1^{'} < f_1$	$A_2 < f_1^{'} < f_1 < B_2$[45]
		$f_1^{'} < f_1 < A_2 < B_2$	Never Exist	$C_2 < B_2 < f_1^{'} < f_1$	$A_2 < C_2 < f_1^{'} < f_1 < B_2$ or $A_2 < f_1^{'} < f_1 < C_2 < B_2$[46]
3.	$\tilde{d}_{1t+1} = f_1^{'}$	$B_3 < f_1^{'} < f_1 < A_3$	$B_3 < A_3 < f_1^{'} < f_1 < C_3$	$B_3 < A_3 < C_3 < f_1^{'} < f_1$	Never Exist

* Proof is available based on request.

Table 5: Change in the Ranges of Existence of Steady-State Equilibria When Reserve Requirements are Set to Zero

Sub-set		Range for ***Type 1***	Range for ***Type2***	Range for ***Type 3***	Range for ***Type 4***
1.	$\tilde{d}_{1t+1} = f_1^{'} - f_0$	Reduce	Reduce	Widen	Never Exist
2.	$\tilde{d}_{1t+1} = f_1^{'} - d_{2t+1}$	Unclear	Reduce	Widen	Unclear
		Unclear	Never Exist	Widen	Unclear
3.	$\tilde{d}_{1t+1} = f_1^{'}$	Reduce	Reduce	Widen	Never Exist

[45] The range for the interior equilibrium to exist under the condition when $B_2<C_2$.
[46] The range for the interior equilibrium to exist under the condition when $B_2>C_2$.

* Proof is available based on request.

Table 6: Change in the Ranges of Existence of Steady-State Equilibria
When Increase the Corresponding Monetary Policy Parameter σ

Sub-set		Range for ***Type 1***	Range for ***Type2***	Range for ***Type 3***	Range for ***Type 4***
1.	$\tilde{d}_{1t+1} = f_1^{'} - f_0$	Reduce	Reduce	Widen	Never Exist
2.	$\tilde{d}_{1t+1} = f_1^{'} - d_{2t+1}$	Widen	Unclear	Widen	Reduce
		Widen	Never Exist	Widen	Reduce
3.	$\tilde{d}_{1t+1} = f_1^{'}$	Reduce	Reduce	Widen	Never Exist

* Proof is available based on request.

Figure 1: Case 1 - Continuum of Equilibria

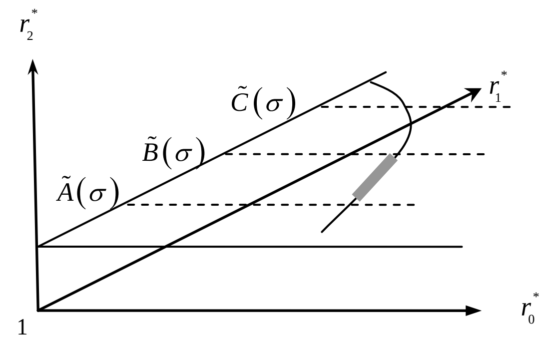


Figure 2: The Structure of the Dynamic System
Floating Exchange Rates

$$\tilde{z}_{t-1} \;\Big|\; \rightarrow \underbrace{z_t \rightarrow \left(\tilde{\tau}_t, q_t, \tilde{b}_t^*, k_t\right)}_{\text{The Core}} \rightarrow \underbrace{\left(\tilde{d}_{0,t,j}, d_{1,t+1,j}, d_{2,t+1,j}\right) \rightarrow \left(\tilde{c}_{1,t,j}, c_{2,t+1,j}\right)}_{\text{Case } j, \text{ for } j=1,2,3}$$

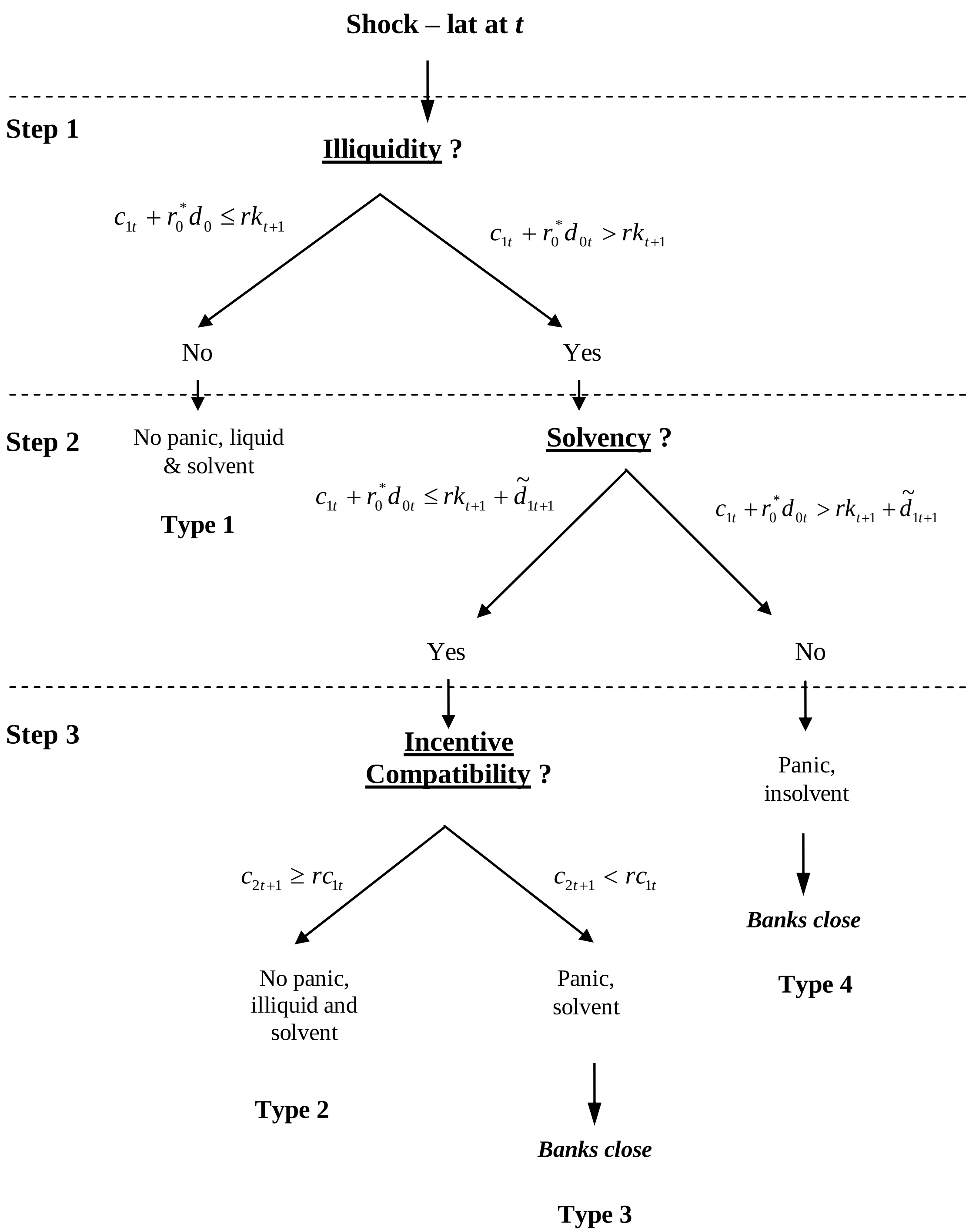
t-1

Figure 3: The Structure of the Dynamic System
Fixed Exchange Rates

$$\overline{\tau}_{t-1} \;\Big|\; \rightarrow \underbrace{\overline{\tau}_t \rightarrow \left(\overline{z}_t, \overline{q}_t, \overline{b}_t^*, \overline{k}_t\right)}_{\text{The Core}} \rightarrow \underbrace{\left(\overline{d}_{0,t,j}, \overline{d}_{1,t+1,j}, \overline{d}_{2,t+1,j}\right) \rightarrow \left(\overline{c}_{1,t,j}, \overline{c}_{2,t+1,j}\right)}_{\text{Case } j, \text{ for } j=1,2,3}$$

t-1

Figure 4. The Sequential Checking Mechanism



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Technical Appendix

1. Comparing Alternative Transfers-Schemes

In this Appendix, we consider the case where the government gives transfers to young agents claiming to be of the impatient type after individuals learn their types. Notice that it is not possible to verify types directly, since they are private information.

1.1 The Model

In the situation where the government gives the transfers to young agents claiming to be of the impatient type after agents learn their types, we can write the conditions for a general equilibrium as

$$k_{t+1} \le d_{0,t+1} + d_{2,t+1} + (1-\phi_f - \phi_d)\cdot w \tag{A.1.1}$$

$$\lambda \cdot c_{1,t} + r_0^* \cdot d_{0t} \le r \cdot l_t + d_{1,t+1} + \lambda \cdot \tau_t \tag{A.1.2}$$

$$(1-\lambda)\cdot c_{2,t+1} + r_2^* \cdot d_{2,t+1} + r_1^* \cdot d_{1,t+1} \le R\cdot(k_t - l_t) + \phi_d \cdot \left(\frac{p_t}{p_{t+1}}\right)\cdot w + \phi_f \left(\frac{1}{1+\sigma^*}\right)\cdot w \tag{A.1.3}$$

$$R > \frac{p_t}{p_{t+1}} \rightarrow \frac{M_t}{p_t} = \phi_d \cdot w \tag{A.1.4}$$

$$R > \frac{p_t^*}{p_{t+1}^*} = \frac{e_{t+1}}{e_t}\cdot\frac{p_t}{p_{t+1}} \rightarrow \frac{e_t \cdot Q_t}{p_t} = \phi_f \cdot w \tag{A.1.5}$$

$$GBC: \lambda \cdot \tau_t = \frac{M_t - M_{t-1}}{p_t} \tag{A.1.6}$$

1.2 Existence of Steady-State Equilibria under Floating Exchange Rates

When the monetary authority gives the transfers to young agents claiming to be of the impatient type, we observe the following:

$$z_{t+1} = z_t = \phi_d \cdot w \tag{A.1.7}$$

$$\lambda \cdot \tau_t = \left(\frac{M_t}{p_t}\right)\cdot\left(\frac{\sigma}{1+\sigma}\right) = z_t \cdot \left(\frac{\sigma}{1+\sigma}\right) = \phi_d \cdot w \cdot \left(\frac{\sigma}{1+\sigma}\right) \tag{A.1.8}$$

$$\frac{p_t}{p_{t+1}} = \left(\frac{M_t}{\phi_d \cdot w}\right) = \left(\frac{1}{1+\sigma}\right) \tag{A.1.9}$$

We present these results in Table A.1 below.

Table A.1: Existence of Steady- State Equilibria with Transfers to Young Impatient Agents, $l_t = 0$ A Floating Exchange Rate Regime

Type of Equilibria	Amount Borrowed	$r_0^* > 1$ and $r_1^* < r_2^* = R$
No intra-period foreign debt	$d_2 = f_0$ $d_0 = 0$ $d_1 = f_1 - f_0$	We can observe one of two cases: 1. Equilibria exist for all relevant parameter values, or 2. Equilibria exist only for values of σ sufficiently high.
Interior solution	$d_2 > 0$ $d_0 = f_0 - d_2$ $d_1 = f_1 - d_2$	Equilibria exist only when σ is neither too low nor too high
No long-term foreign debt	$d_2 = 0$ $d_0 = f_0$ $d_1 = f_1$	We can observe one of three cases: 1. Equilibria do not exist, or 2. Equilibria exist only for values of σ that are neither too low nor too high, or 3. Equilibria exist only for values of σ sufficiently high.

1.3 Existence of Steady-State Equilibria under Fixed Exchange Rates

Table A.2 presents the results for situations where the monetary authority gives transfers to young agents claiming to be of the impatient type.

Table A.2: Existence of Steady-State Equilibria with Transfers to Young Impatient Agents
A Fixed Exchange Rate Regime

Type of Equilibria	Amount Borrowed	$r_0^* > 1$ and $r_1^* < r_2^* = R$
No intra-period foreign debt	$d_2 = f_0$ $d_0 = 0$ $d_1 = f_1 - f_0$	We can observe one of three cases: 1. Equilibria do not exist, or, 2. Equilibria exist for values of σ^* that are neither too low nor too high, or. 3. Equilibria exist only for values of σ^* sufficiently high.
Interior solution	$d_2 > 0$ $d_0 = f_0 - d_2$ $d_1 = f_1 - d_2$	Equilibria exist for values of σ^* that are neither too low nor too high.
No long-term foreign debt	$d_2 = 0$ $d_0 = f_0$ $d_1 = f_1$	We can observe one of three cases: 1. Equilibria do not exist, or, 2. Equilibria exist for values of σ^* that are neither too low nor too high, or 3. Equilibria exist only for values of σ^* sufficiently high.

2. Existence of the right-corner solution for $d_2 = 0$

From the F.O.C. of the banks' problem, these set of equilibria {d_2=0, d_0=f_0, d_1=f_1} will exist when $r_1^* > A$, $r_1^* > B$, and $r_1^* \le C$. We discuss the subset as follows:

I. C<A whenever $(1-\lambda)\left(\frac{R}{r}\right)f_1 < \{A_1\}^{\frac{1}{2}} - \{C_1\}^{\frac{1}{2}}$

II. Otherwise, A<C

a) B<C when $a > -\left\{\frac{(f_1 - r_0^* \cdot f_0)\cdot\left[\lambda Rf_0 - (1-\lambda)rf_1 + (C_1)^{1/2}\right]}{(1-2\lambda)f_1 - r_0^* f_0}\right\}$

and $(1-2\lambda)f_1 - r_0^* f_0 > 0$

b) Otherwise, C<B when $\lambda > 1/2$, where

$$a \equiv \lambda\left[R(1-\phi_d-\phi_f) + \phi_d\left(\frac{1}{1+\sigma}\right) + \phi_f\left(\frac{1}{1+\sigma^*}\right)\right](w+\tau)$$

$$A \equiv \left(\frac{1}{2\lambda f_1}\right)\left(a + \lambda Rf_0 - (1-\lambda)\left(\frac{R}{r}\right)f_1 + \{A_1\}^{\frac{1}{2}}\right),\; B \equiv \left(\frac{a}{f_1 - r_0^* f_0}\right),\; C \equiv \left(\frac{1}{2\lambda f_1}\right)\left(a + \lambda Rf_0 - (1-\lambda)rf_1 + \{C_1\}^{\frac{1}{2}}\right),$$

$$A_1 \equiv \left((1-\lambda)\left(\frac{R}{r}\right)f_1 - \lambda Rf_0 - a\right)^2 + 4\lambda(1-\lambda)r\left(\frac{R}{r}\right)^2 f_1 f_0,\; C_1 \equiv ((1-\lambda)rf_1 - \lambda Rf_0 - a)^2 + 4\lambda(1-\lambda)rRf_1 f_0$$

We summarize the existence of these stationary equilibria in Table A.3.

Table A.3. The Existence of the Left-corner (d_2=0) Equilibria under Floating

Subsets		Existence of equilibria
I. C<A		No equilibria
II. A<C	a) B<C	Equilibria exist when $\max(A,B) < r_1^* \le C$
	b) C<B	No equilibria

Since the monetary authority can choose to hold foreign reserves under both exchange rate regimes, the government budget constraint τ_t is the same under floating and fixed arrangements. Also, we know the policy parameter enter into the condition for existence through transfers τ. These equilibria d_2=0 will exist when $\max(A,B) < r_1^* \le C$ under both regimes. The difference between the two exchange rate regimes is the monetary policy. Under floating, we consider the case when domestic money supply grows at a constant rate, where $\frac{M_t}{M_{t+1}} = \frac{p_t}{p_{t+1}} = \frac{1}{1+\sigma}$. On the other hand, money supply is tied with the exchange rate under fixed regime, where $\frac{p_t}{p_{t+1}} = \frac{p_t^*}{p_{t+1}^*} = \frac{1}{1+\sigma^*}$.

3. Existence of Equilibria under Floating Exchange Rate Regimes

3.1 Conditions for Existence in Case 1. We must define first the following expression: $\hat{\varepsilon} \equiv \lambda \cdot w \cdot \{ r_2^* \cdot (1-\phi_d-\phi_f) + [\phi_f/(1+\sigma^*)]\} - r_1^* \cdot (f_1 - f_0) \cdot \{1-\phi_d \cdot [1+\theta \cdot (\tilde{r}-1)]\}$. Next, we enumerate the two mutually exclusive conditions needed for this type of stationary equilibrium to exist under floating:

Condition 1: $\hat{\varepsilon} > 0$ must hold.

Condition 2: when $\hat{\varepsilon} < 0$, $\sigma < \hat{\sigma} \equiv [\tilde{\varepsilon} + \lambda \cdot \phi_d \cdot w/(-\varepsilon)]$ must hold.

Ceteris paribus, values of r_1^* that are sufficiently low increase the scope for which and are typically associated with situations where the relative cost of the debt-instrument $\widehat{d_{0,1}}$ is perceived as high. Thus, when banks choose their portfolio, it is in their interest to avoid this expensive instrument. Condition 2 instead is a statement about the rate of domestic money growth: low enough values of σ imply high return on the domestic currency reserves held by banks, and thus, banks are willing to borrow arbitrarily large amounts of the long-term instrument, which matures in the same date.

3.2 Conditions for Existence in Case 2. There is only one condition for the existence of the interior solution equilibria:

Condition 3: $r_0^* > r$ and $r_1^* > r$

Notice that Condition 3 always holds, since $r < 1$. The amount borrowed of long-term debt in equilibrium is

$$\widehat{d_{2,2}} = \left[r_2^* \cdot (1-\phi_d-\phi_f) + \left(\frac{\phi_d}{1+\sigma}\right) + \left(\frac{\phi_f}{1+\sigma^*}\right)\right] \cdot \frac{\lambda \cdot (w+\tau)}{(r_2^* - r_1^*)} - r_1^* \cdot (f_1 - r_0^* \cdot f_0) \qquad \text{(A.3.1)}$$

while, it is evident that $0 < \widehat{d_{2,2}} < f_0$, $\widehat{d_{0,2}} = f_0 - \widehat{d_{2,2}} > 0$ and $\widehat{d_{1,2}} = f_1 - \widehat{d_{2,2}} > 0$ hold.

3.3 Conditions for Existence in Case 3. The conditions for existence in this case are fairly complex and algebra-intensive, and the reader interested in details can find them in Appendix 2. Before we proceed, we must define the following expressions:

$A \equiv \left(\frac{\bar{a} + \lambda \cdot r_2^* \cdot f_0 - (1-\lambda) \cdot \left(f_1 \cdot r_2^* / r \right) + (\breve{a})^{1/2}}{2 \cdot \lambda \cdot f_1} \right)$, $B \equiv \left(\frac{\bar{a}}{f_1 - r_0^* \cdot f_0} \right)$, $C \equiv \left[\frac{\bar{a} + \lambda \cdot r_2^* \cdot f_0 - (1-\lambda) \cdot r \cdot f_1 + (\check{c})^{1/2}}{2 \cdot \lambda \cdot f_1} \right]$,

$\bar{a} \equiv \left[r_2^* \cdot (1 - \phi_d - \phi_f) + \left(\frac{\phi_d}{1+\sigma} \right) + \left(\frac{\phi_f}{1+\sigma^*} \right) \right] \cdot \lambda \cdot (w + \tau)$, $\check{c} \equiv \left[(1-\lambda) \cdot r \cdot f_1 - \lambda \cdot r_2^* \cdot f_0 - \bar{a} \right]^2 + 4 \cdot \lambda \cdot (1-\lambda) \cdot r \cdot r_2^* \cdot f_1 \cdot f_0$, and $\breve{a} \equiv \left[(1-\lambda) \cdot \left(f_1 \cdot r_2^* / r \right) - \lambda \cdot r_2^* \cdot f_0 - \bar{a} \right]^2 + 4 \cdot \lambda \cdot (1-\lambda) \cdot \left[\left(r_2^* \right)^2 / r \right] \cdot f_1 \cdot f_0$. Equilibria in this case exist when the following condition holds:

<u>Condition 4:</u> $\max\{A, B\} < r_1^* \leq C$ holds.

4. Local Stability Analysis of Foreign Debt in Equilibria of Case 2 – Floating Exchange Rates

- <u>Dynamics and the world inflation rate.</u> The interaction with changing values of σ^* introduces interesting variations to the sequence of dynamic properties that we describe below.
  a) For low enough values of the world inflation rate such that $\sigma^* \in (-1, 0)$ the sequence becomes: (+)sink, (+)complex-stable, (-)sink and (-)saddle, thus increasing the scope for determinacy and stable fluctuations with respect to the *baseline scenario*.
  b) As σ^* increases gradually, the economy goes back to the *baseline sequence*, but the scope for determinacy decrease gradually as well.
  c) When σ^* increases from inflation crises values to hyperinflation, the sequence becomes: (+)sink, (+)complex-stable, (-)complex-stable and (-)complex unstable, eliminating the scope for determinacy.
  d) Next, for even higher values of σ^*, the scope for complex eigenvalues with negative real parts decreases until it eventually disappears, and the sequence becomes: (+)sink, (+)complex-stable, but with almost explosive values of the discriminant which translate into very large cyclical fluctuations.

  In conclusion, low enough rates of foreign inflation contribute to the goal of increased stability and determinacy of dynamic equilibria, as one might expect, but coupled with endogenously-arising volatility.
- <u>Dynamics and world interest rates.</u> In this case, we proceed by changing gradually the pair $(\tilde{r}, R = r_2^*)$ [47] to study how the dynamic sequence changes. We describe our findings below.
  a) For $\tilde{r} = R = 1$ (i.e. zero net returns,) the associated dynamic sequence is: (+)sink, (+)complex-stable, (-)complex-stable, (-)sink and (-)saddle, where the scope for determinacy dominates. Thus, this combination of returns eliminates the scope for diverging non-cyclical fluctuations.
  b) As $(\tilde{r}, R)$ increase gradually, the dynamic sequence converges to the *baseline sequence*.
  c) As the pair $(\tilde{r}, R)$ continues to increase, the scope for complex eigenvalues increases significantly together with the source, while the scope for determinacy eventually disappears completely. Thus, high foreign interest rates promote instability and fairly large cyclical and non-cyclical fluctuations.
- <u>Dynamics and reserve requirements.</u> We proceed by increasing gradually both reserve requirements, with the condition that $\phi_d = \phi_f < 0.5$. The reader will notice that the reserve requirements play the role of stabilizing dynamic equilibria. We find:

[47] The interest rates (r_0^*, r_1^*) change accordingly, but we do not report them here since they have no effect on the dynamics in this case.

a) For $\phi_d = \phi_f$ close enough to zero the dynamic sequence consists of: (+)sink followed by (+)complex-stable, but with very high values of the discriminant, and thus very large cyclical fluctuations.

b) As $\phi_d = \phi_f$ increase gradually, the scope for (+)complex-stable decreases, giving rise instead to (-)complex, (-)source and a (-)saddle until converging to the *baseline sequence*.

c) High enough values of $\phi_d = \phi_f$ reduce the scope for complex eigenvalues and (-) source, which gives rise to a (-)sink and a significantly larger range of (-) saddle. Thus, high and binding reserve requirements promote stability and determinacy along the dynamic paths.

- Dynamics and backing of the money supply. Different fractions of the money supply to be backed by the monetary authority introduce interesting changes in the dynamic sequence, which we describe below. The reader will notice that the policy parameter θ plays the role of de-stabilizing of dynamic equilibria.

a) When $\theta = 0$, the order of the dynamic system decreases to a first order nonlinear difference equation. As a result, for low enough values of σ, the eigenvalue is positive and stable, promoting monotonic dynamics. As σ increases, the eigenvalue becomes negative but stable and damped oscillations are observed along dynamic paths. Finally, for values of σ that are high enough, the negative eigenvalue turns unstable, and explosive oscillations take place.

b) For very low, non-zero values of θ, the system turns into a second order nonlinear difference equation where the following dynamic sequence arises: (+)sink, (+)complex-stable, (-)complex-stable, (-)sink and (-)saddle. This sequence displays both stability and determinacy along dynamic paths.

c) As the backing of the domestic money supply increases, the dynamic sequence converges to the *baseline sequence*, where instability is observed.

d) For values of θ close enough to 1, the scope for complex eigenvalues increases significantly, eliminating gradually the (-) source as well as the (-) saddle. Dynamic equilibria are indeterminate and large cyclical and non-cyclical fluctuations dominate along dynamic paths.

- Dynamics and combinations of reserve requirements and backing of the domestic money supply. We now evaluate alternative combinations of reserve requirements and backing of the domestic money supply, in order to find the combination that promotes stability and determinacy. We now describe our findings.

a) Low $(\phi_d = \phi_f, \theta)$. Under this combination, the dynamic sequence becomes: (+)sink, (+)complex-stable, (-)complex-stable, (-)complex-unstable, and (-) source. Thus, the scope for instability and fluctuations dominates.

b) High $(\phi_d = \phi_f, \theta)$. Under this combination, the dynamic sequence turns into: (+)sink, (+)complex-stable, (-)complex-stable, (-)complex-unstable, (-) source and (-)saddle. Thus, the scope for determinacy increases.

c) Low $\phi_d = \phi_f$ and high θ. The dynamic sequence turns out to be: (+)sink, (+)complex-stable, (-)complex-stable, and (-)complex-unstable. Equilibria are indeterminate and display cyclical fluctuations that become unstable.

d) High $\phi_d = \phi_f$ and low θ. The dynamic sequence in this case is (+)sink, (+)complex-stable, (-)complex-stable, (-)sink and (-)saddle. Thus, this combination of policy parameters promotes both the stability and determinacy of dynamic equilibria.

5. Existence of Equilibria under a Hard Peg

Conditions for Existence in Case 1. In this case, we must define first the expression $\overline{\varepsilon} \equiv \lambda \cdot w \cdot r_2^* \cdot \left(1-\phi_d-\phi_f\right) - r_1^* \cdot \left(f_1 - f_0\right) \cdot \left\{1-\phi_d \cdot \left[1+\theta \cdot \left(\tilde{r}-1\right)\right]\right\}$. Next, we establish the condition for existence under a fixed exchange regime.

Condition 5: $\overline{\varepsilon} > 0$ must hold.

Condition 5 is more likely to hold for values of r_1^* and/or θ that are sufficiently low. The intuition behind this finding is very simple: when $\overline{d}_{1,1}$ is relative cheap, banks will borrow less of its short-term substitute $\overline{d}_{0,1}$ The latter may condition banks to borrow more of the long-term debt, $\overline{d}_{2,1}$.

Conditions for Existence in Case 2. Under a fixed exchange rate regime, the following condition must hold for an interior solution to exist equilibria:

Condition 6: $r_0^* > r$ and $r_1^* > r$ must hold.

The reader may notice that Condition 6 is identical to Condition 3 with floating exchange rates. It always holds $\forall\left(r_0^*, r_1^*, r_2^*\right) \gg 1$, since $r < 1$.

Conditions for Existence in Case 3. Under a fixed exchange rate regime, we must substitute the expression $\bar{a}$ by $\bar{\bar{a}} \equiv \left\{r_2^* \cdot (1-\phi_d-\phi_f) + \left[\left(\phi_d+\phi_f\right) / \left(1+\sigma^*\right)\right]\right\} \cdot \lambda \cdot \left(w+\overline{\tau}\right)$. Of course, the expressions $\left(A, B, C, \breve{a}, \check{c}\right)$ must be modified accordingly, giving rise instead to $\left(\overline{A}, \overline{B}, \overline{C}, \bar{\breve{a}}, \bar{\check{c}}\right)$. Thus, the condition for the existence of equilibria in Case 3 becomes:

Condition 7: $\max\left(\overline{A}, \overline{B}\right) < r_1^* \leq \overline{C}$ must hold.

Thus, Condition 7 requires the debt instrument $\overline{d}_{1,3}$ to be sufficiently expensive, so that banks are conditioned to acquire more of the instrument $\overline{d}_{0,3}$ instead. It follows, in the limit, that $\overline{d}_{0,3} = f_0$ and $\overline{d}_{2,3} = 0$ in equilibrium. Table 6 summarizes the conditions for existence as well as how the scope for existence depends on the world inflation rate σ^*.

6. Stability properties regarding other parameters under a Hard Peg

- Rates of return $\left(\tilde{r}, R\right)$: When $\tilde{r} = R = r_2^* = r_0^* = r_1^* = 1$ (zero net returns,) the scope for stability almost disappears, while the scopes for very large unstable fluctuations and very large monotonic divergence dominate. A very small increase in the returns eliminates the scope for monotonic divergence while the scope for stable monotonic dynamics dominates. As the returns increase, the dynamics converges toward the *baseline sequence*. In the limit, the maximum eigenvalue approaches to zero from the right. Thus, higher returns seem to stabilize dynamic equilibria.
- Number of impatient agents λ: $\forall \lambda \in \left(0,1\right)$, no monotonic divergence is observed, and only small variations around the *baseline sequence* take place. The scope for large unstable fluctuations increases with λ, at the expense of damped oscillations. The scope for stable monotonic dynamics dominates and remains unchanged. The maximum eigenvalue increases with λ, but it is always less than 1.
- Reserve Requirements $\phi_d = \phi_f$: no monotonic divergence is observed $\forall \phi_d = \phi_f < 0.5$ [48], and the dynamic sequence deviates only slightly from the baseline sequence. As $\phi_d = \phi_f$ increase, the scope and size of unstable fluctuations augment. Thus, fairly large oscillations may be observed around the steady-state while the scope

[48] Recall that we require $\left(1-\phi_d-\phi_f\right) > 0$.

for stable oscillations decreases. The scope for monotonic convergence dominates and it remains unaffected. Thus, binding reserve requirements preserve monotonic stability.

- Dynamics and θ : the policy parameter θ is a stabilizer for long-term debt dynamics. The value $\theta = 0$ eliminates monotonic dynamics, and, specifically, fairly large unstable fluctuations may arise. As θ increases and crosses a threshold value[49], stable monotonic dynamics arise, and the scope for large unstable fluctuations decreases. $\theta = 1$ alone is not enough to produce monotonic divergence, nor is it $\lambda \to 1^-$. However, coupling $\theta = 1$ with a high enough value of λ increases the maximum eigenvalue beyond 1 for high values of σ^* [50].
- Dynamics and combinations of $\phi_d = \phi_f$ and θ : We now compare four alternative combinations of reserve requirements and backing of the domestic money supply against the baseline sequence, in order to find the combination that promotes monotonic stability.
  a) Low $(\phi_d = \phi_f, \theta.)$ Under this combination, range and size of unstable fluctuations decreases. At the same time, the scope of stable fluctuations increases at the expense of the scope of stable monotonic dynamics, but the latter still dominates.
  b) High $(\phi_d = \phi_f, \theta.)$ This combination enlarges the range and size of unstable fluctuations. At the same time, the scope of stable fluctuations shrinks while the scope of stable monotonic dynamics raises and it still dominates.
  c) Low $\phi_d = \phi_f$ and high θ . This combination reduces significantly the range and size of unstable fluctuations together with a fall in the scope for stable fluctuations. Moreover, the range of stable monotonic dynamics increases dramatically and it dominates the general scope of dynamics. Thus, this combination of policy parameters promotes stable monotonic dynamics.
  d) High $\phi_d = \phi_f$ and low θ. This combination adds to the range and size of unstable fluctuations as well as to the scope for stable fluctuations. Consequently, the range of stable monotonic dynamics suffers, falling dramatically. Moreover, stable fluctuations dominate the general scope of dynamics. Notice that this policy had stabilizing effects under floating exchange rates but now it is the opposite.

7. The Functions in Table 4

$$A_1 \equiv (1+\lambda r) f_0 + \lambda r \cdot (1-\phi_d-\phi_f)(w+\tau_t) \quad \text{(A7.1)}$$

$$B_1 \equiv \left(\frac{1}{\lambda}\right) f_1 - \left(\frac{1}{\lambda}+r-1\right) f_0 - r(1-\phi_d-\phi_f)(w+\tau_t) \quad \text{(A7.2)}$$

$$C_1 \equiv f_0 + \left(\frac{\lambda}{\lambda r_1^* + (1-\lambda) r}\right)\left[R(1-\phi_d-\phi_f)(w+\tau_t) + RDC + RFC\right] \quad \text{(A7.3)}$$

$$RDC \equiv (w+\tau_t)\phi_d\left(\frac{p_t}{p_{t+1}}\right) \quad \text{(A7.4)}$$

$$RFC \equiv (w+\tau_t)\phi_f\left(\frac{1}{1+\sigma^*}\right) \quad \text{(A7.5)}$$

$$A_2 \equiv r_0^* f_0 - \left(\frac{1}{y}\right)\left((r_0^*-r) f_0 + (w+\tau_t)\left\{\left[\left(\frac{1}{\lambda}-1\right) r_0^* - \frac{1}{\lambda}\right]\left(\frac{\lambda}{r_2^*-r_1^*}\right)\left[(1-\phi_d-\phi_f) + \frac{\phi_d p_t}{p_{t+1}} + \frac{\phi_f}{1+\sigma^*}\right] - r(1-\phi_d-\phi_f)\right\}\right) \quad \text{(A7.6)}$$

$$y \equiv \lambda\left\{1 + \left[\frac{1}{r_0^*} - (1-\lambda)\right] R\right\} \quad \text{(A7.7)}$$

$$B_2 \equiv \left[\frac{1}{\lambda} - \left(\frac{1}{\lambda}-1\right)(r_2^*-r_1^*)\right](f_1 - r_0^* f_0) + (r_0^*-r) f_0 + ((1-\lambda) r_0^* - r)(1-\phi_d-\phi_f)(w+\tau_t) + \left(\frac{1-\lambda}{r_1^*}\right)(RDC+RFC) \quad \text{(A7.8)}$$

$$C_2 \equiv r_0^* f_0 + \frac{r}{y_1}\left[R(1-\phi_d-\phi_f)(w+\tau_t) + RDC + RFC\right] \quad \text{(A7.9)}$$

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[49] In our simulations, this threshold value was $\theta = 0.037$.
[50] The values in our simulations were $\lambda = 0.766$ and $\sigma^* > 23.000$.

$$y_1 \equiv \left(\frac{r_1^*}{1-\lambda} + \frac{r}{\lambda}\right)\left\{1 - \lambda r_1^*\left(r_0^* - 1\right)\right\} \tag{A7.10}$$

$$A_3 \equiv \left(\frac{r_0^*}{1-\lambda} + \frac{r}{\lambda}\right) f_0 + \left(\frac{r}{\lambda}\right) \cdot \left(1 - \phi_d - \phi_f\right)\left(w + \tau_t\right) \tag{A7.11}$$

$$B_3 \equiv \left(\frac{1}{\lambda}\right) f_1 + \left(\left(1 - \frac{1}{\lambda}\right) r_0^* - r\right) f_0 - r\left(1 - \phi_d - \phi_f\right)\left(w + \tau_t\right) \tag{A7.12}$$

$$C_3 \equiv r_0^* f_0 + \left[\frac{\lambda}{\lambda r_1^* + (1-\lambda) r}\right]\left[R\left(1 - \phi_d - \phi_f\right)\left(w + \tau_t\right) + RDC + RFC\right] \tag{A7.13}$$

Notice that for different exchange rate regimes, the corresponding government transfers are different, so as the rate of return of domestic and foreign currency reserves.